Unit - 1 Set, Relation and Function

* Task : 1

3 Which of the following are null set?

(1) $A = \{x : x \in R$ and x is a solution of $x^2 + 2 = 0\}$

$A = \{\emptyset\}$

Here, A is null set.

(2) $B = \{x : x \in Z$ and x is a solution of $x - 3 = 0\}$

$A = \{3\}$

Here, B is not null set.

(3) $C = \{x : x \in Z$ and x is a solution of $x^2 - 2 = 0\}$

$A = \{\emptyset\}$

Here, C is null set.

4 Write the power set of the following sets.

(1) $A = \{x : x \in R$ and $x^2 + 7 = 0\}$

$A = \emptyset$

(2) $B = \{y : y \in N$ and $1 \leq y \leq 3\}$

$B = \{1, 2, 3\}$

Power Set $= \{ \{1,2,3\}, \{1,2\}, \{1,3\},$
P(B) $\{2,3\}, \{2,1\}, \{\emptyset\},$
$\{3,1\}, \{3,2\} \}$

5 Let $A = \{x : x$ is a even natural number less then or equal to $10\}$

$B = \{x : x$ is an odd natural less then or equal to $10\}$

$A = \{2, 4, 6, 8, 10\}$
$B = \{1, 3, 5, 7, 9\}$

(i) $A - B$

$A - B = \{2, 4, 6, 8, 10\} - \{1, 3, 5, 7, 9\}$
$= \{\cancel{\ }\} A$

(ii) $B - A$

$B - A = \{1, 3, 5, 7, 9\} - \{2, 4, 6, 8, 10\}$
$= \cancel{\{\}}\ B$

(iii) $A - B = B - A$?

Here $A - B = \emptyset$ and
$B - A = \emptyset$

So, $A - B = B - A$

1 For $A = \{a, b, \{a, c\}, \emptyset\}$ determine the following set.

(1) $A - \{\{a, b\}\} = A$

(2) $A - \emptyset = A$

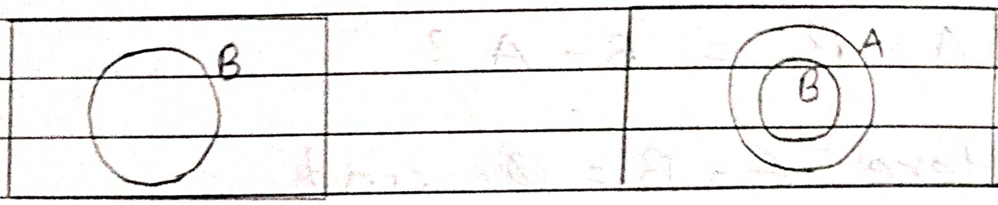
(3) $\{a, c\} - A = \{\cancel{a}, b, \cancel{\emptyset}\}\ \emptyset$

(4) $\{\emptyset\} - A = \cancel{A}\ \emptyset$

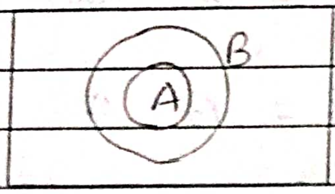
(5) $\{a\} - A = \{\cancel{b}, \{a, c\}, \cancel{\emptyset}\}\ \emptyset$

2 Draw Venn Diagram For the Following condition.

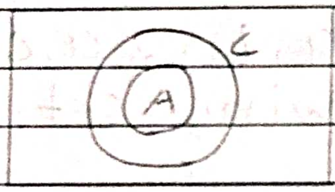
(1) $(A \cup B) \subseteq B$ and $B \subseteq A$



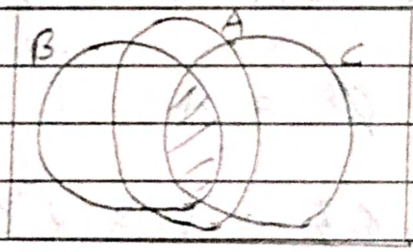
(2) $A \subseteq B \Rightarrow$



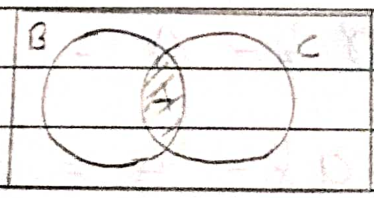
-> $A \subseteq C \Rightarrow$



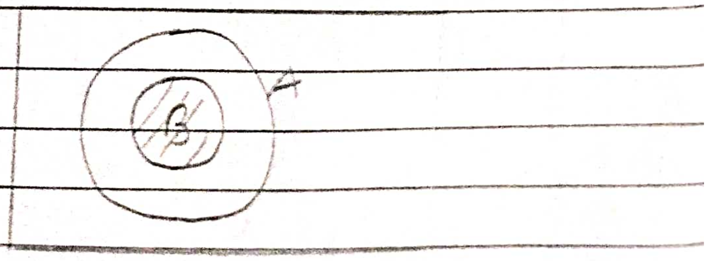
-> $(B \cap C) \subseteq A \Rightarrow$



-> $A \subseteq (B \cap C) \Rightarrow$



(3) $A \cap B$ when $B \subset A \Rightarrow$



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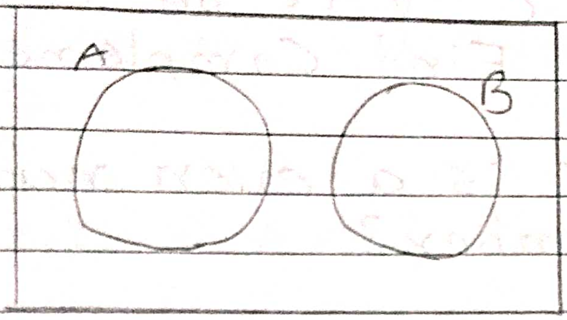
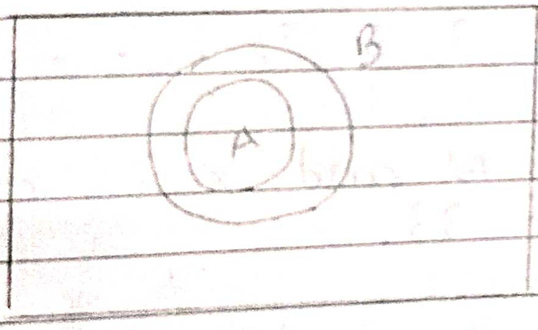
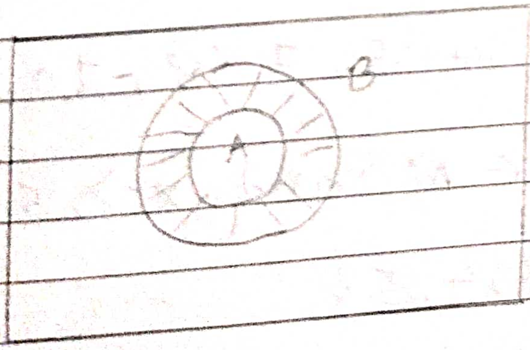
(4) A U B when A and B are disjoint set.



(5) A - B and B - A when A ⊂ B



A - B and A ⊂ B



B - A and A ⊂ B

* Task : 2

1 Let N be the universal set and A, B, C, D is be its subset given by, Find Complement.

(1) $A = \{x : x$ is a even natural number$\}$

Here, $N = \{1, 2, 3, 4, 5, 6, ...\}$

For $A = \{2, 4, 6, 8, 10, ...\}$

$A' = \{1, 3, 5, 7, 9, ...\}$

(2) $A = \{x : x \in N$ and x is multiple of $3\}$

$A = \{3, 6, 9, 12, 15, 18, ...\}$

$A' = \{1, 2, 4, 5, 7, 8, 10, ...\}$

(3) $A = \{x : x \in N$ and $x \geq 5\}$

$A = \{1, 2, 3, 4, 5\}$

$A' = \{6, 7, 8, 9, 10, ...\}$

(4) $D = \{x : x \in N \text{ and } x \leq 10\}$

$D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$D' = \{11, 12, 13, 14, 15, 16, 17, ---\}$

3 If $A \cup B = A \cap B$ show that $A = B$.

Here, ~~$x \in A \cup B$~~

If $x \in A$ and $x \in B$

If $x \in B \Rightarrow x \in A \cup B$ and $x \in A \cap B$
such that $x \in A$
$\therefore B \subseteq A$

$\therefore$

If $x \in A \rightarrow x \in A \cup B$ and
$x \in A \cap B$
$\therefore x \in B$
$\therefore A \subseteq B$

Hence, $A = B$

So, $A \cup B = A \cap B$ and set $A = B$
is proven.

4 Prove that $A - (B \cup C) = (A - B) \cap (A - C)$

Here L.H.S $= A - (B \cup C)$

L.H.S $= A - (B \cup C)$

Here, $x \in A$ and $x \notin (B \cup C)$

$\therefore$ $x \in A$ and $(x \notin B$ and $x \notin C)$

$\therefore$ $(x \in A$ and $x \notin B)$ and $(x \in A$ and $x \in C)$

$\therefore$ $x \in (A - B)$ and $x \in (A - C)$

$\therefore$ $x \in (A - B) \cap (A - C)$

$=$ R.H.S

Hence,

$A - (B \cup C) = (A - B) \cap (A - C)$

2 If $B \subset A$, prove that,

(1) $B \cup C \subset A \cup C$

Let, $A = \{a, b, c, d, e\}$
$B = \{b, c, d\}$

$\therefore B \subset A$
$C = \{f, g, h\}$

$\rightarrow B \cup C = \{b, c, d, f, g, h\}$ — 1

$\rightarrow A \cup C = \{a, b, c, d, e, f, g, h\}$ — 2

By eq^n 1 and 2,

$B \cup C \subset A \cup C$

Hence, $B \cup C \subset A \cup C$ is prove.

(2) $B \cap C \subset A \cap C$

Let, $A = \{a, b, c, d, e\}$
$B = \{b, c, d)$

$\therefore B \subset A$

$C = \{b, c, d, e, g\}$

$\rightarrow B \cap C = \{b, c, d\}$ — 1

→ $A \cap C = \{b, c, d, e\}$ — (2)

By eq^n 1 and 2

$B \cap C \subset A \cap C$

Hence, $B \cap C \subset A \cap C$ is prove.

{2. Draw Venn diagram for the following set}

* Task : 3

1 Find the domain and range of the relation R where R is define in (a) and (b)

(a) $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 10\}$, aRb if and only if $2a = b$

Set $R_1 = \{(a, b) / 2a = b\}$

$R_1 = \{(1, 2), (5, 10)\}$

(b) $A = \{1, 2, 3, 4\} = B$, aRb if and only if $a+b = 5$

Set $R_2 = \{(a, b) / a+b = 5\}$

$R_2 = \{(1, 4), (2, 3), (4, 1), (3, 2)\}$

2 For each of the following relation on $A = \{1, 2, 3, 4\}$ determine whether its reflexive, symmetric or transitive.

(a) $R = \{(1, 4), (4, 1)\}$

Here, Given relation R is be a Symmetric relation if and only if

$aRb \Rightarrow bRa \quad , \quad a, b \in R$

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$\therefore (1,4) \Rightarrow (4,1)$

So that R is a Symmetric Relation.

(b) $R = \{(1,1)\}$

Here R be a Reflexive relation if

$aRa \Rightarrow aRa, \; a \in R$

Here for $\forall A \cancel{\notin} \in R$,

So that R is ~~not~~ be a reflexive relation.

(c) $R = \{(1,1), (2,2), (3,3), (4,4), (2,3), (3,2)\}$

→ Here R be a Reflexive relation if $aRa, \; \forall a \in R$

$\therefore (1,1) \in R$ for $\forall (a,a) \in R$

Such that R is a reflexive relation.

→ Here R be a Symmetric relation if $aRb \Rightarrow bRa, \; \forall a, b \in R$

$\therefore (a,b) \in R, \; (b,a) \in R$

Such that R is a Symmetric Relation.

(d) $R = \{(1,3),(1,4)\}$

→ Reflexive Relation:

Here R be a reflexive Relation if aRa ; $a \in R$

$\therefore (a,a) \notin R$

So that, R is not Reflexive Relation.

→ Symmetric Relation:

Here R be a symmetric relation if ~~aRB~~
$aRb \Rightarrow bRa \ , \forall a,b \in R$

$(a,b) \in R$, $(b,a) \notin R$.

So that, R is not Symmetric Relation.

→ Transitive Relation:

Here, R be a transitive relation if $aRb, bRc \Rightarrow aRc, \forall a,b,c \in R$

$\therefore (a,b) \in R$, $(b,c) \notin R$, $(a,c) \notin R$

So that, R is not transitive Relation.

Hence, R is not be any Relation.

3 Let $X = \{1,2,3,4,5,6,7\}$ and $R = \{\langle x, y\rangle /$ $x-y$ is divisible by $3\}$, then show that R is an equivalence relation.

→ Here Given Set $X = \{1,2,3,4,5,6,7\}$ and $R = \{\langle x, y\rangle / x-y$ is divied by $3\}$

$R = \{(1,1), (1,4), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (5,2), (2,5), (6,3), (3,6), (7,4), (4,7), \cancel{(2,5)} (5,6)$ & $, (6,5)\}$

For Equivalence relation we have to check three relation,

(a) Reflexive Relation:

Let R be a Reflexive relation, if aRa, $\forall a \in R$

$\therefore (a,a) \in R, \forall a \in R$

So, that R is Reflexive Relation.

(b) Symmetric Relation:

Let R be a Symmetric relation
if $aRb \Rightarrow bRa$, $a, b \in R$

$\therefore (a, b) \in R \rightarrow (b, a) \in R$
$\forall (a, b) \in R$

So, that R is a Symmetric Relation.

(c) Transmitive Relation:

Let R be a transitive Relation
if aRb, $bRc \Rightarrow aRc$, $a, b, c \in R$

$\therefore (a, b) \in R$, $(b, a) \in R$
$\therefore (a, c) \in R$

So, that R is a transitive Relation.

Hence, R is follow this three Relation.

So, R is a Equivalence Relation.

5 Let $A = \{1, 2, 3, 4, 5\}$ and Let R be a relation on A define,

$$R = \{(1,3), (2,1), (2,2), (2,5), (3,4), (4,3), (4,4), (5,1), (5,3)\}$$

then Compute R^2, R^3, R^{-1}, $R \circ R^{-1}$, $R^{-1} \circ R$.

→ Let $R = \{(1,3), (2,1), (2,2), (2,5), (3,4), (4,3), (5,1), (5,3), (4,4)\}$

$R^{-1} = \{(3,1), (1,2), (2,2), (5,2), (4,4), (4,3), (3,4), (1,5), (3,5)\}$

$R \circ R = R^2 = \{(1,4), (2,3), (2,1), (2,2), (2,5), (3,4), (3,3), (4,4), (4,3), (5,3), (5,4)\}$

$R \circ R^2 = R^3 = \{(1,4), (2,3), (2,1), (2,2), (2,5), (3,4), (3,3), (4,4), (4,3), (5,3), (5,4)\}$

$R \circ R^{-1} = \{(1,1), (1,4), (1,5), (2,2), (3,3), (3,4), (4,1), (4,4), (4,5), (4,3), (5,1), (5,4), (5,5), (5,2)\}$

$R^{-1} o R = \{ (3,1), (1,1), (1,2), (1,5), (2,1), (2,2), (2,5), (5,1), (5,2), (4,4), (3,3), (3,4), (4,3), (3,2) \}$

~~4) Let X =~~

4 On the set Z of all integer, define the relation R by

$$R = \{ (a,b) \in Z \times Z \,/\, a-b \text{ divide by } 5 \}$$

Show that R is an equivalance relation.

Here, Given Relation.

$$R = \{ (a,b) \in Z \times Z \,/\, a-b \text{ divide by } 5 \}$$

For Equivalence relation, we have to check three relation,

(a) Reflexive Relation:

R be a Reflexive relation, if aRa, $\cancel{a \forall}$ $a \in R$,

$$\therefore (a,a) \in R \rightarrow a-a = 0/5 = 0 \in R$$

So, R is Reflexive Relation.

(b) Symmetric Relation:

R be a Symmetric relation,

if $aRb \Rightarrow bRa$, $a, b \in R$

$\therefore (a, b) \in R \rightarrow (a-b)/5 \in R$
$\therefore (b, a) \in R \rightarrow (b-a)/5 \in R$

So, that R is Symmetric.

(c) Transitive Relation:

R be a Transitive Relation
if aRb, $bRc \Rightarrow aRc$, $\forall a, b, c \in R$

$\therefore a-b/5$, $b-c/5$

$\therefore \frac{(a-b)}{5} + (b-c)/5$

$\therefore (a-c)/5 \rightarrow aRc$

So, that R is Transitive.

Hence, R is follow this three relaction.

Such that R is a Equivalence Relation.

* Task : 4

1 $A = \{1, 2, 3, 4\}$ and $B = \{p, q, r, s\}$ and $R = \{(1,p), (1,q), (1,r), (2,q), (2,r), (2,s)\}$ then Find matrix relation M_R and draw Arrow diagram.

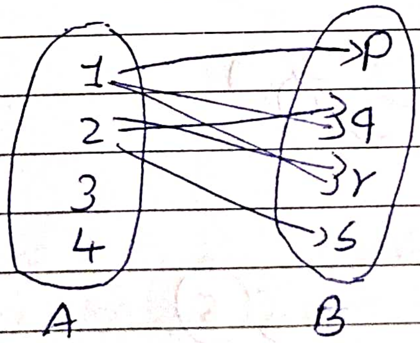
-> Here, Given $R = \{(1,p), (1,q), (1,r), (2,q), (2,r), (2,s)\}$

- Matrix Relation

$M_R =$

A \ B	p	q	r	s
1	1	1	1	0
2	0	1	1	1
3	0	0	0	0
4	0	0	0	0

- Arrow Diagram:



2 Let $A = \{1, 4, 5\}$ and $R = \{(1,4), (1,5), (4,1), (4,4), (5,5)\}$.

Then Find matrix M_R and draw a diagraph for R.

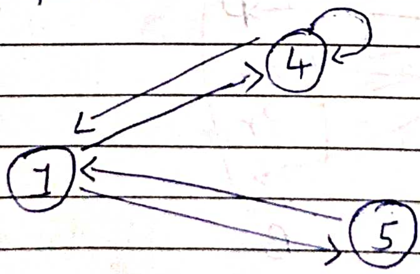
Here, Given Relation.

$$R = \{(1,4), (1,5), (4,1), (4,4), (5,5)\}$$

→ Matrix Relation

$$M_R = \begin{array}{c|ccc} A \backslash A & 1 & 4 & 5 \\ \hline 1 & 0 & 1 & 1 \\ 4 & 1 & 1 & 0 \\ 5 & 0 & 0 & 1 \end{array}$$

→ Diagraph:



3 Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 4, 6, 8, 9\}$
Let R be a relation from set A to set B and defined as $_aR_b$ if and

only if $b = a^2$. Then Find matrix relation M_R and drow Arrow diagram.

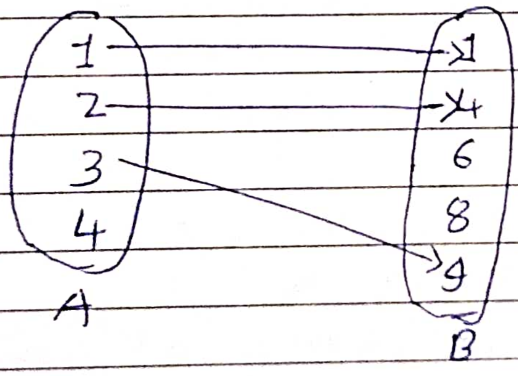
→ Here, Given Relation,

$$R = \{(1,1),(2,4),(3,9)\}$$

Matrix Relation

$$M_R = \begin{array}{c|ccccc} A \backslash B & 1 & 4 & 6 & 8 & 9 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{array}$$

→

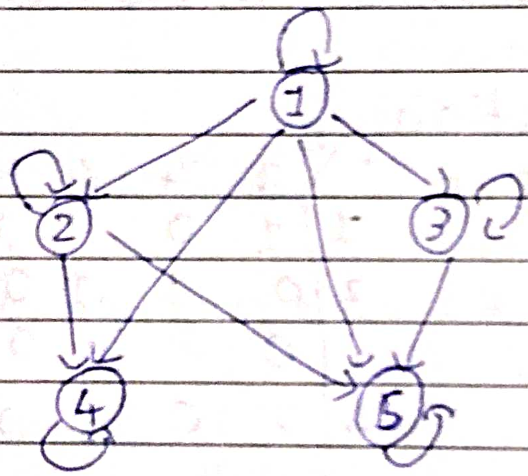


4 Let $A = \{1,2,3,4,6\}$ be a set and R be a relation on set A, defind as $_aR_b$ if and only if a is multiple of b and Draw diagraph.

→ Here Given ~~relation~~ set $A = \{1,2,3,4,6\}$

Relation R = {(1,1), (1,2), (1,3),
(1,4), (1,6), (2,4),
(2,6), (3,6), (2,2),
(3,3), (4,4), (6,6)}

→ Diagraph:



* Task: 5

1 Determine which of the following function are one-one, on-to or both.

(a) $F: N \rightarrow Z - \{0\}$ defined by $f(n) = -n$ for all $n \in N$.

Here, Given function $f(n) = -n$
and $n_1, n_2 \in N$

→ For One-One function,

$$f(n_1) = f(n_2)$$
$$-n_1 = -n_2$$
$$n_1 = n_2$$

So, $f(n)$ is One-One function.

→ For On-to function,

$$f(n) = -n$$
$$y = -n$$
$$\therefore n = -y$$
$$\therefore f(-y) = -(-y)$$
$$\therefore f(n) = y$$

So, $f(n)$ is also On-to function.

Hence, $F(n)$ is One-One and Onto function.

(b) $F: Z \rightarrow Z$, defined by $F(x) = X - 4$ for all $x \in Z$.

Here, Given function $F(x) = x - 4$ and $x_1, x_2 \in Z$

→ For One-One function,

$$F(x_1) = F(x_2)$$
$$\therefore x_1 - 4 = x_2 - 4$$
$$\therefore x_1 = x_2$$

So, $F(x)$ is One-One function.

→ For Onto function,

$$F(x) = x - 4$$
$$y = x - 4$$
$$\therefore x = y + 4$$
$$\therefore F(x) = \cancel{y} - 4 + 4$$
$$\therefore F(x) = y$$

So, $F(x)$ is Onto function.

Hence, $F(x)$ is One-One and Onto function.

(c) $F : R \to R$ define by $F(x) = |x| + x$ for all $x \in R$.

Here, Given function $F(x) = |x| + x$

$$F(x_1) = \begin{cases} 0, & x_1 \leq 0 \\ 2x_1, & x_1 > 0 \end{cases}$$

Let $F(x_1) = 2x_1$ when $x_1 > 0$
$F(x_1) = 0$ when $x_1 \leq 0$

So, $F(x)$ is not One-One function.

For Onto, $F_1(x_1) = 0 \; ; \; x \leq 0$
$\therefore \; Y = 0 = F_1(x)$

$F_2(x) = 2x \; ; \; x > 0$
$\therefore \; Y = 2x = F_2(x)$

$\therefore \; F_1(x) \neq F_2(x)$

thus, $F(x)$ is not onto or One-One function.

(b) $F: R \to R$ define by $F(x) = x^3$ for all $x \in R$.

Here, Given function $f(x) = x^3$ and $x_1, x_2 \in R$

-> For One-One function,

$$\therefore f(x_1) = f(x_2)$$
$$\therefore x_1^3 = x_2^3$$
$$\therefore x_1 = x_2$$

So, $f(x)$ is One-One function.

-> For Onto Function,

$$f(x) = y$$
$$\therefore f(x) = x^3$$
$$\therefore x^3 = y$$

$$\therefore x = y^{+1/3}$$

$$\therefore f(x) = x^3$$
$$= (y^{+1/3})^3$$

$$\therefore f(x) = y$$

So, $f(x)$ is Onto function.

Hence, $f(x)$ is One-One and Onto function.

2 Let $f: R \to R$ defined by $f(x) = x^2 - 4x$ then find $I_m(f)$ is f onto? Is f One-One.

Here, Given function $f(x) = x^2 - 4x$

$x_1, x_2 \in R$

-> For One-One Function,

$$f(x_1) = f(x_2)$$

$$\therefore x_1^2 - 4x_1 = x_2^2 - 4x_2$$

$$\therefore x_1 = x_2$$

$f(x)$ is One-One function.

-> For Onto Function,

$$\therefore f(x) = y$$

$$\therefore x^2 - 4x = y$$

3 Let $f : Q \to Q$ be the function defined by $f(x) = 3x + 4$ for all $x \in Q$ then find the inverse of f if it exist.

Here, Given function $f(x) = 3x + 4$ and $x_1, x_2 \in Q$

→ For One-One Function,

$$\therefore f(x_1) = f(x_2)$$
$$\therefore 3x_1 + 4 = 3x_2 + 4$$
$$\therefore x_1 = x_2$$

So, $f(x)$ is One-One function.

→ For Onto function,

$\therefore f(x) = y$

$\therefore 3x+4 = y$

$\cancel{\therefore x = 3x}$

$\therefore x = \frac{y-4}{3}$

$\because f(x) = 3\left(\frac{y-4}{3}\right)+4$

$\therefore f(x) = y$

So, f(x) is Onto function.

-> Inverse of f(x) is exist.

$\therefore f^{-1} : Q \to Q$

$\therefore f^{-1} = \frac{x-4}{3}$

4 Let $f: N \to N$ be defined by $f(n) = n+3$, for all $n \in N$ then show that f is one-one but ~~not~~ onto. or both.

Here, Given Function $f(n) = n+3$
and $n_1, n_2 \in N$

-> For One-One Function,

$\therefore F(n_1) = F(n_2)$
$\therefore n_1 + 3 = n_2 + 3$

$\therefore n_1 = n_2$

So, $F(n)$ is One-One function.

-> For Onto Function,

$\therefore F(n) = y$
$\therefore y = n + 3$
$\therefore n = y - 3$

$\therefore F(n) = y - 3 + 3$

$\therefore F(n) = y$

So, $F(n)$ is Onto function.

Hence, $F(n)$ is One-One and Onto Function.