

Unit - 2 : Lattices

* Task : 1

1 Define Poset with Example.

Let a relation define on non-empty set X is partially ordered r^n then, this set is called "Partially Order Set".

Poset are denoted by $\langle X, \leq \rangle$

Ex. Suppose A is any non-empty set and show that $\langle PCA, \leq \rangle$ is partially ordered r^n .

→ Here, given set is all the subset of A which is PCA and r^n , " \leq ".

(a) Reflexive:

$\forall A_1 \in PCA$,
We write $A_1 \leq A_1$

So, " \leq " is Reflexive.

(b) Anti symmetric:

$$\forall A_1, A_2 \in PCA,$$

Suppose $A_1 \leq A_2$ and $A_2 \leq A_1$,
then always $A_1 = A_2$.

" \leq " is anti symmetric.

(c) Transitive:

$$\forall A_1, A_2, A_3 \in PCA,$$

If $A_1 \leq A_2$ and $A_2 \leq A_3$ then
 $A_1 \leq A_3$ is possible.

" \leq " is transitive.

Relation " \leq " is Partially Ordered set on R^n .

Hence, $\langle PCA, \leq \rangle$ is Poset.

2) Consider the set Z of integer. Define $a R b$ if there is a positive integer r such that $b = a^r$, is R a Poset on Z ?

→ Here, Given set = Z and
 $a R b$ if $b = a^r, r \in Z^+$

For Poset on Z must be satisfy this three properties.

(1) Reflexive: $\forall a \in Z$

if $a R a$ then $a = a^r$ for $r = 1 \in Z^+$
 $\therefore R$ is Reflexive.

(2) Anti-symmetric: $\forall a, b \in Z$

if $a R b$ then $a = b^r$ or $b = a^r$
 for $r = 1 \in Z^+$

$\therefore R$ is anti-symmetric.

(3) Transitive: $\forall a, b, c \in Z$

if $a R b$ and $b R c$ then,

$$a = b^r, c = b^s$$

$$\therefore c = (a^r)^s \text{ for } r = 1 \in Z^+$$

$$\therefore a R c$$

$\therefore R$ is transitive.

Hence, $b = r^n$ is a Poset of Z .

3 Let $N = \{1, 2, 3, \dots\}$ be ordered by \mathbb{D} . State whether each of the following subset of N are linearly ordered.

(a) $\{24, 2, 6\}$

$$R = \{(24, 2), (2, 24), (24, 6), (6, 24), (2, 6), (6, 2)\}$$

Here, every pair is follows $\frac{a}{b}$ or $b/a \in N$ set.

\therefore Subset of N are totally Ordered.

(b) $\{3, 15, 5\}$

$$R = \{(3, 15), (3, 5), (15, 5), (15, 3), (5, 3), (5, 15)\}$$

Here, every pair is not follows $\frac{a}{b}$ or $b/a \notin N$ set.

So, subset of N are not totally Ordered.

(d) $\{2, 8, 32, 4\}$

$$R = \{(2, 8), (2, 32), (2, 4), (8, 32), (8, 4), (8, 2), (32, 4), (32, 2), (32, 8), (4, 2), (4, 8), (4, 32)\}$$

Here, every pair is not follows a/b or $b/a \in N$.

So, N subset of N are not linearly Ordered.

(c) $\{1, 2, 3, \dots\}$

$$R = \{(1, 2), (1, 3), (1, 4), \dots\}$$

Here, every pair is not follows a/b or $b/a \in N$.

So, subset of N are not linearly Ordered.

(e) $\{7\}$

Here, Given set is not containt other element.

So, subset of N are linearly Ordered.

(F) $\{15, 5, 30\}$

$$R = \{(15, 5), (15, 30), (5, 30), (5, 15), (30, 15), (30, 5)\}$$

Here, every pair follows
 a/b or $b/a \in \mathbb{N}$.

So, subset of \mathbb{N} is linearly Ordered.

4 Show that $\langle P(X), \subseteq \rangle$ is a poset where X is non empty set and $P(X)$ is a poset set of X .

Here Given set is all the subset of X .

For Poset, on X must be follow this three condition.

(a) Reflexive: For $\forall x \subseteq P(X)$,

$$\therefore x, \subseteq x,$$

$\therefore "$ \subseteq " is Reflexive.

(b) Antisymmetric: For $\forall x_1, x_2 \in P(X)$.

if $x_1 \subseteq x_2$ then $x_2 \subseteq x_1$ and always $x_1 = x_2$.

" \subseteq " is antisymmetric.

(c) Transitive: $\forall x_1, x_2, x_3 \in P(A)$

if $x_1 \subseteq x_2$, and $x_2 \subseteq x_3$ then
for subset of power set is
possible $x_1 \subseteq x_3$.

" \subseteq " is Transitive.

Hence, $\langle P(A), \subseteq \rangle$ is Poset.

5 Which of the following are Posets?

(a) $\langle \mathbb{Z}, = \rangle$

For Poset,

(i) Reflexive: $\forall a \in \mathbb{Z}, a = a$

\therefore " $=$ " is Reflexive.

(ii) Antisymmetric: $\forall a, b \in \mathbb{Z},$

$\therefore a = b$ then $b = a$.

\therefore " $=$ " is Antisymmetric.

(iii) Transitive: $\forall a, b, c \in \mathbb{Z}$

if $a = b$ and $b = c$ then always
 $a = c$

\therefore " $=$ " is Transitive.

Hence, $\langle \mathbb{Z}, = \rangle$ is Poset.

(b) $\langle \mathbb{Z}, \neq \rangle$

For Poset,

(i) Reflexive: $\forall a \in \mathbb{Z}$, always
 $a = a$.

We can not do $a \neq a$.

So, $\langle \mathbb{Z}, \neq \rangle$ is not Poset.

(c) $\langle \mathbb{Z}, > \rangle$

For Poset,

Reflexive: $\forall a \in \mathbb{Z}$, always $a = a$,
there is not $a > a$.

So, $\langle \mathbb{Z}, > \rangle$ is not Poset.

(d) $\langle \mathbb{Z}, < \rangle$

For Poset,

Reflexive: $\forall a \in \mathbb{Z}$, always $a = a$.
We can not write $a < a$.

So, $\langle \mathbb{Z}, < \rangle$ is not Poset.

6 Show that $\langle \{1, 3, 3^2, 3^3, \dots\}, \mathcal{D} \rangle$ is a chain.

Here, Given set $R = \{1, 3, 3^2, 3^3, \dots\}$

For Poset, there are three Properties must be follow.

(i) Reflexive: $\forall a \in R$,

$\therefore a$ is always \mathcal{D} by a .

\therefore " \mathcal{D} " is Reflexive.

(ii) Transitive: $\forall a, b, c \in R$.

if a^r / b^r and b^r / c^r then,

a^r / c^r is must be \mathcal{D}
by

So, " \mathcal{D} " is Transitive.

ciii) Antisymmetric:

$\forall a^r, b^r \in \mathcal{Z}$

Suppose, $a^r \mid b^r$ and $b^r = a^r$
then $a^r = b^r \in \mathcal{N}$

\therefore " \mathcal{D} " is Antisymmetric.

Hence, $\langle \{1, 3, 3^2, \dots\}, \mathcal{D} \rangle$ is
a chain.

* Task: 2

1 Draw the following Mass Diagram.

(a) $\langle S_{30}, D \rangle$

$$S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

Cover of all the elements,

$$1 \rightarrow 2, 3, 5$$

$$2 \rightarrow 6, 10$$

$$3 \rightarrow 6, 15$$

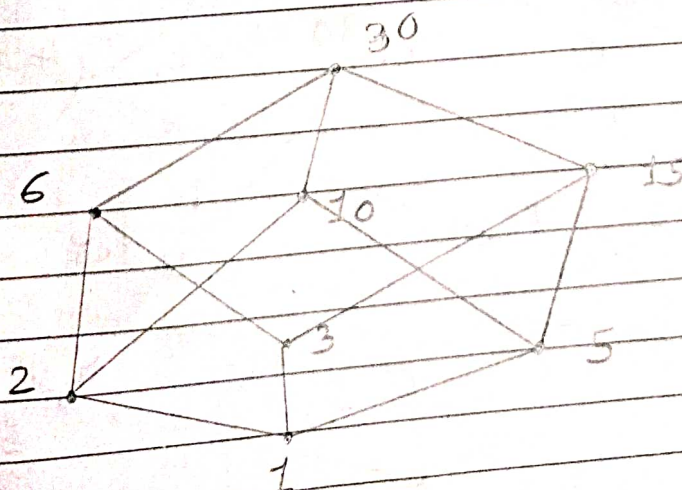
$$5 \rightarrow 10, 15$$

$$6 \rightarrow 30$$

$$10 \rightarrow 30$$

$$15 \rightarrow 30$$

$$30 \rightarrow \emptyset$$



(b) $\langle S_{90}, D \rangle$

$S_{90} = \{1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90\}$

Cover of all the element,

1 \rightarrow 2, 3, 5

2 \rightarrow 6, 10

3 \rightarrow 6, 9, 15

5 \rightarrow 10, 15

6 \rightarrow 18, 30

9 \rightarrow 18, 45

10 \rightarrow 30

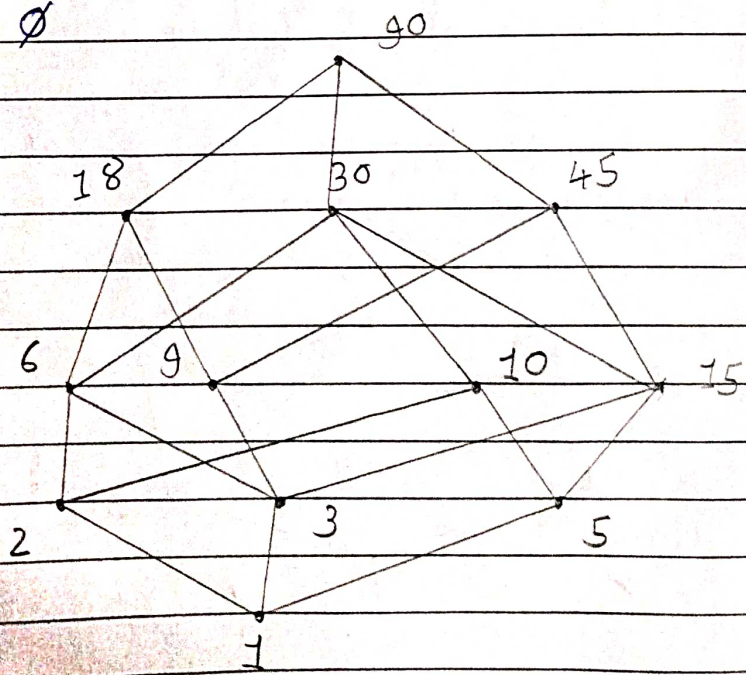
15 \rightarrow 30, 45

18 \rightarrow 90

30 \rightarrow 90

45 \rightarrow 90

90 \rightarrow \emptyset



(c) $\langle P(A), \subseteq \rangle$ where $A = \{a, b, c, d\}$

$$P(A) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{c, d, a\}, \{d, a, b\}, \{a, b, c, d\} \}$$

Cover of all the element,

$$\emptyset \rightarrow \{a\}, \{b\}, \{c\}, \{d\}$$

$$\{a\} \rightarrow \{a, b\}, \{a, c\}, \{a, d\}$$

$$\{b\} \rightarrow \{a, b\}, \{b, d\}, \{b, c\}$$

$$\{c\} \rightarrow \{a, c\}, \{c, d\}, \{b, c\}$$

$$\{d\} \rightarrow \{b, d\}, \{c, d\}, \{a, d\}$$

$$\{a, b\} \rightarrow \{a, b, c\}, \{b, c, d, a\}$$

$$\{a, c\} \rightarrow \{a, b, c\}, \{c, d, a\}$$

$$\{a, d\} \rightarrow \{a, b, d\}, \{a, c, d\}$$

$$\{b, c\} \rightarrow \{a, b, c\}, \{b, c, d\}$$

$$\{b, d\} \rightarrow \{a, b, d\}, \{b, c, d\}$$

$$\{c, d\} \rightarrow \{c, d, a\}, \{b, c, d\}$$

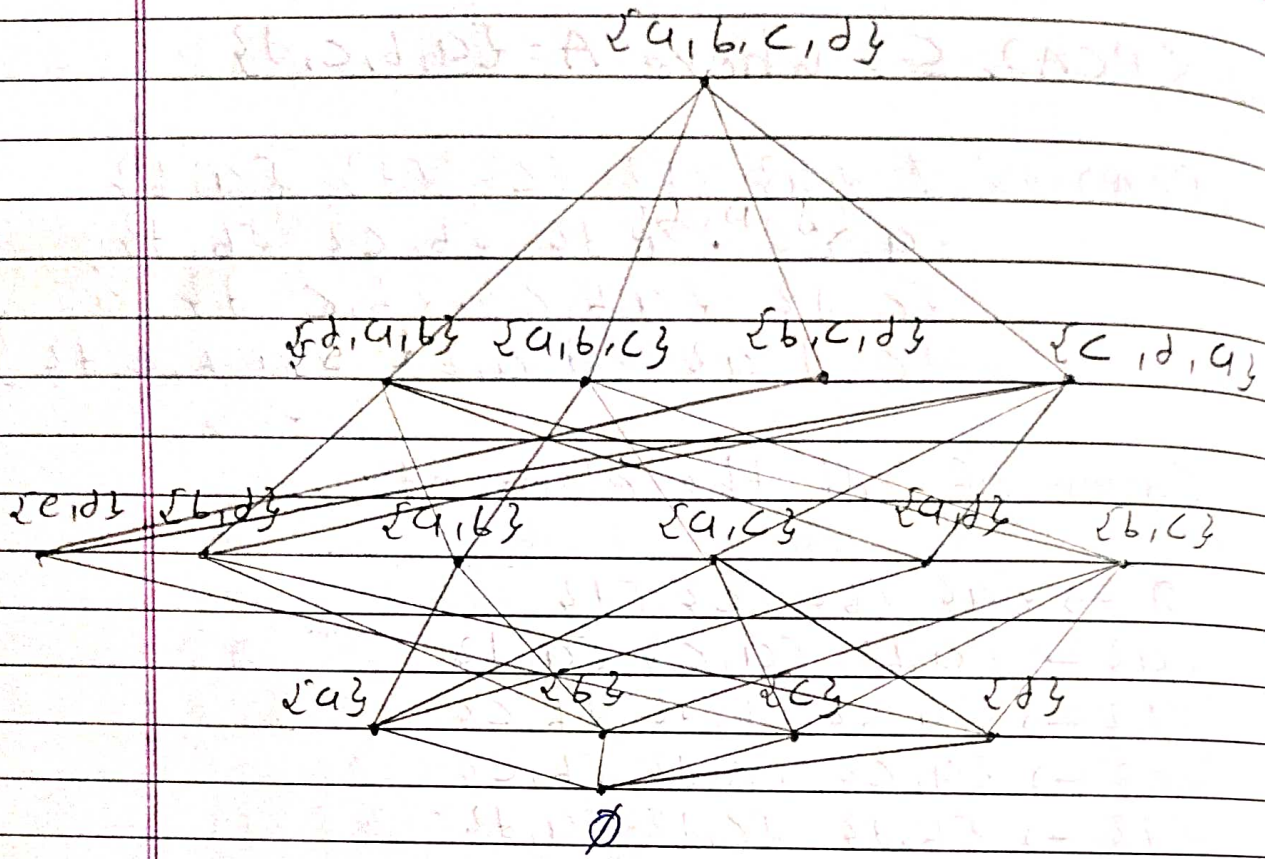
$$\{a, b, c\} \rightarrow A$$

$$\{b, c, d\} \rightarrow A$$

$$\{c, d, a\} \rightarrow A$$

$$\{d, a, b\} \rightarrow A$$

$$\{a, b, c, d\} \rightarrow X$$



(d) $\langle S_{210}, D \rangle$

$$S_{210} = \{1, 2, 3, 5, 6, 7, 10, 21, 30, 35, 42, 70, 105, 210\}$$

Cover of the all the element.

- 1 \rightarrow 2, 3, 5, 7
- 2 \rightarrow 6, 10
- 3 \rightarrow 6, 21
- 5 \rightarrow 10, 35
- 6 \rightarrow 30, 42
- 7 \rightarrow 21, 35
- 10 \rightarrow 30, 70

$$21 \rightarrow 42, 105$$

$$30 \rightarrow 210$$

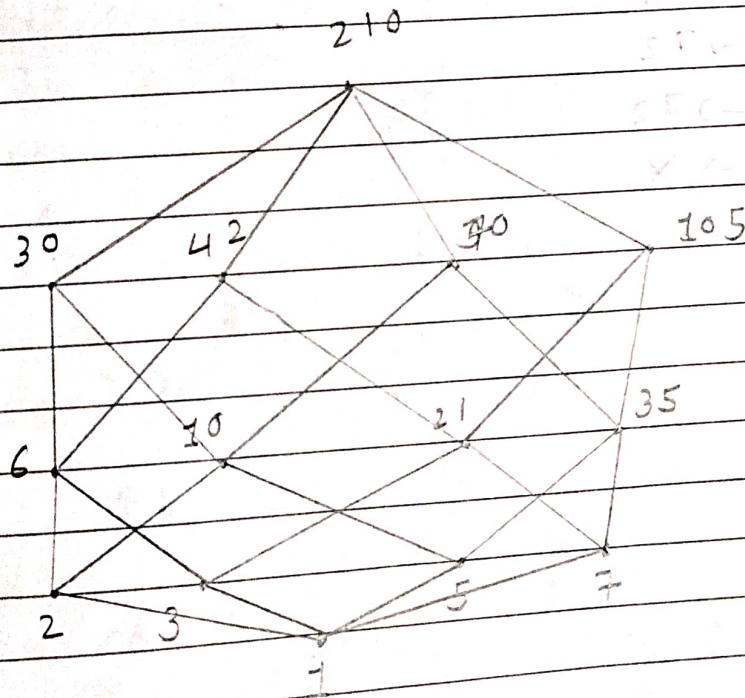
$$35 \rightarrow 70, 105$$

$$42 \rightarrow 210$$

$$70 \rightarrow 210$$

$$105 \rightarrow 210$$

$$210 \rightarrow X$$

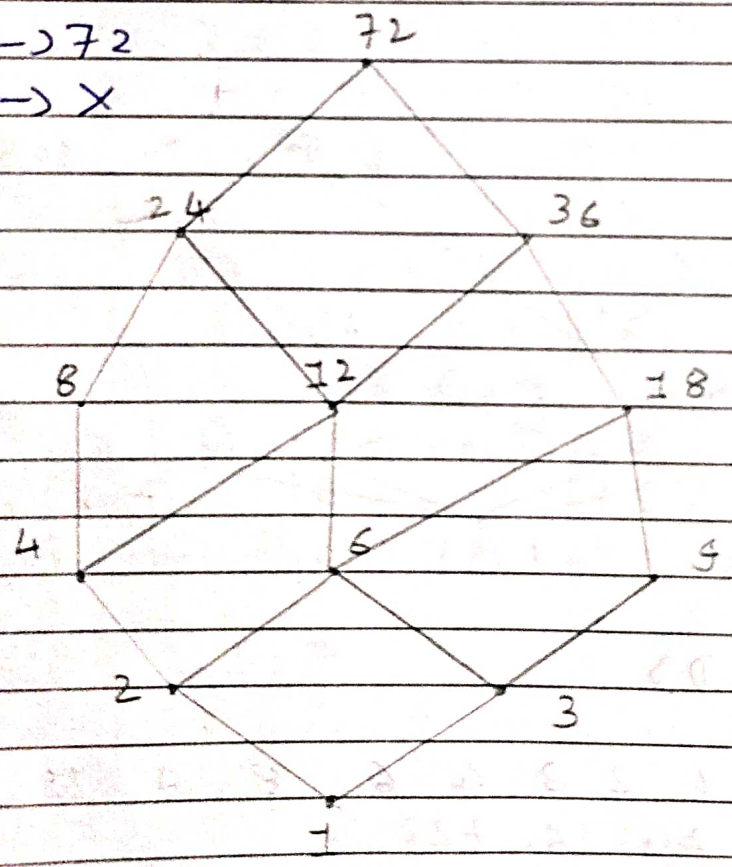


(e) $\langle S_{72}, D \rangle$

$$S_{72} = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$$

Cover of the all element:

- 1 → 2, 3
- 2 → 4, 6
- 3 → 6, 9
- 4 → 8, 12
- 6 → 12, 18
- 8 → 24
- 9 → 18
- 12 → 24, 36
- 18 → 36
- 24 → 72
- 36 → 72
- 72 → X



(g) $\langle S_{24}, D \rangle$

$$S_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$$

Cover of all the element:

$$1 \rightarrow 2, 3$$

$$2 \rightarrow 4, 6$$

$$3 \rightarrow 6$$

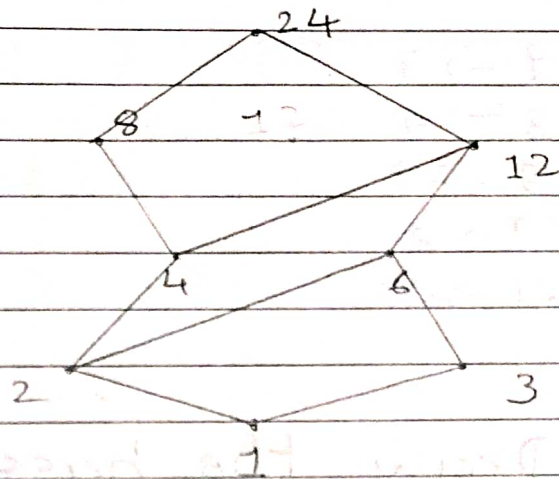
$$4 \rightarrow 8, 12$$

$$6 \rightarrow 12$$

$$8 \rightarrow 24$$

$$12 \rightarrow 24$$

$$24 \rightarrow X$$



(h) $\langle S_{63}, D \rangle$

$$S_{63} = \{1, 3, 7, 9, 21, 63\}$$

Cover of all the element:

$$1 \rightarrow 3, 7$$

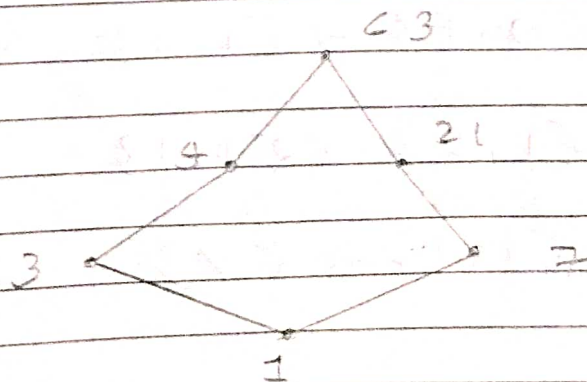
$$3 \rightarrow 9$$

$$7 \rightarrow 21$$

$$9 \rightarrow 63$$

$$21 \rightarrow 63$$

$$63 \rightarrow X$$



(i) $\langle S_{81}, \mathbb{D} \rangle$

$$S_{81} = \{1, 3, 9, 27, 81\}$$

Cover of the all element:

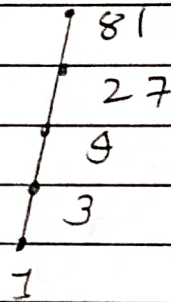
$$1 \rightarrow 3$$

$$3 \rightarrow 9$$

$$9 \rightarrow 27$$

$$27 \rightarrow 81$$

$$81 \rightarrow X$$



2 Draw the hasse diagram of $\langle L^2, \subseteq \rangle$; where $L^2 = L \times L$, $L = \{0, 1\}$.

$$L^2 = \{0, 0\}, \{0, 1\}, \{1, 0\}, \{1, 1\}$$

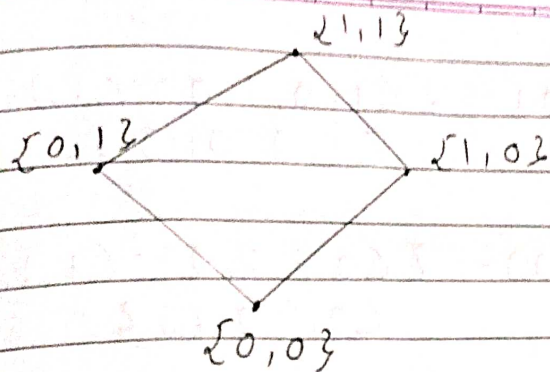
Cover of the all element:

$$\{0, 0\} \rightarrow \{0, 1\}, \{1, 0\}$$

$$\{0, 1\} \rightarrow \{1, 1\}$$

$$\{1, 0\} \rightarrow \{1, 1\}$$

$$\{1, 1\} \rightarrow X$$



3 Draw the Hasse Diagram of $\langle L^4, \leq \rangle$
 where $L^4 = L \times L \times L \times L$ and $L = \{0,1\}$.

$$L^4 = \{ (0,0,0,0), (0,0,1,0), (0,1,0,0), (0,1,1,0), (1,0,0,0), (1,0,1,0), (1,1,0,0), (1,1,1,0), (0,0,0,1), (0,0,1,1), (0,1,0,1), (0,1,1,1), (1,0,0,1), (1,0,1,1), (1,1,0,1), (1,1,1,1) \}$$

Cover of all the element:

$$(0,0,0,0) \rightarrow \{ (0,0,1,0), (0,0,0,1), (1,0,0,0), (0,1,0,0) \}$$

$$(0,0,1,0) \rightarrow \{ (0,1,1,0), (1,0,0,0), (0,0,1,1) \}$$

$$(0,1,0,0) \rightarrow \{ (0,1,1,0), (0,1,0,1), (1,1,0,0) \}$$

$$(0,1,1,0) \rightarrow \{ (0,1,1,1), (1,1,1,0) \}$$

$$(1, 0, 0, 0) \rightarrow \{(1, 0, 1, 0), (1, 1, 0, 0), (1, 0, 0, 1)\}$$

$$(0, 1, 0, 0) \rightarrow \{(0, 1, 1, 0), (1, 1, 0, 0), (0, 1, 0, 1)\}$$

$$(1, 0, 1, 0) \rightarrow \{(1, 1, 1, 0), (1, 0, 1, 1)\}$$

$$(0, 0, 1, 1) \rightarrow \{(0, 1, 1, 1), (1, 0, 1, 1)\}$$

$$(0, 1, 0, 1) \rightarrow \{(0, 1, 1, 1), (1, 1, 0, 1)\}$$

$$(1, 0, 0, 1) \rightarrow \{(1, 0, 1, 1), (1, 1, 0, 1)\}$$

$$(1, 1, 0, 0) \rightarrow \{(1, 1, 1, 0), (1, 1, 0, 1)\}$$

$$(0, 1, 1, 1) \rightarrow (1, 1, 1, 1)$$

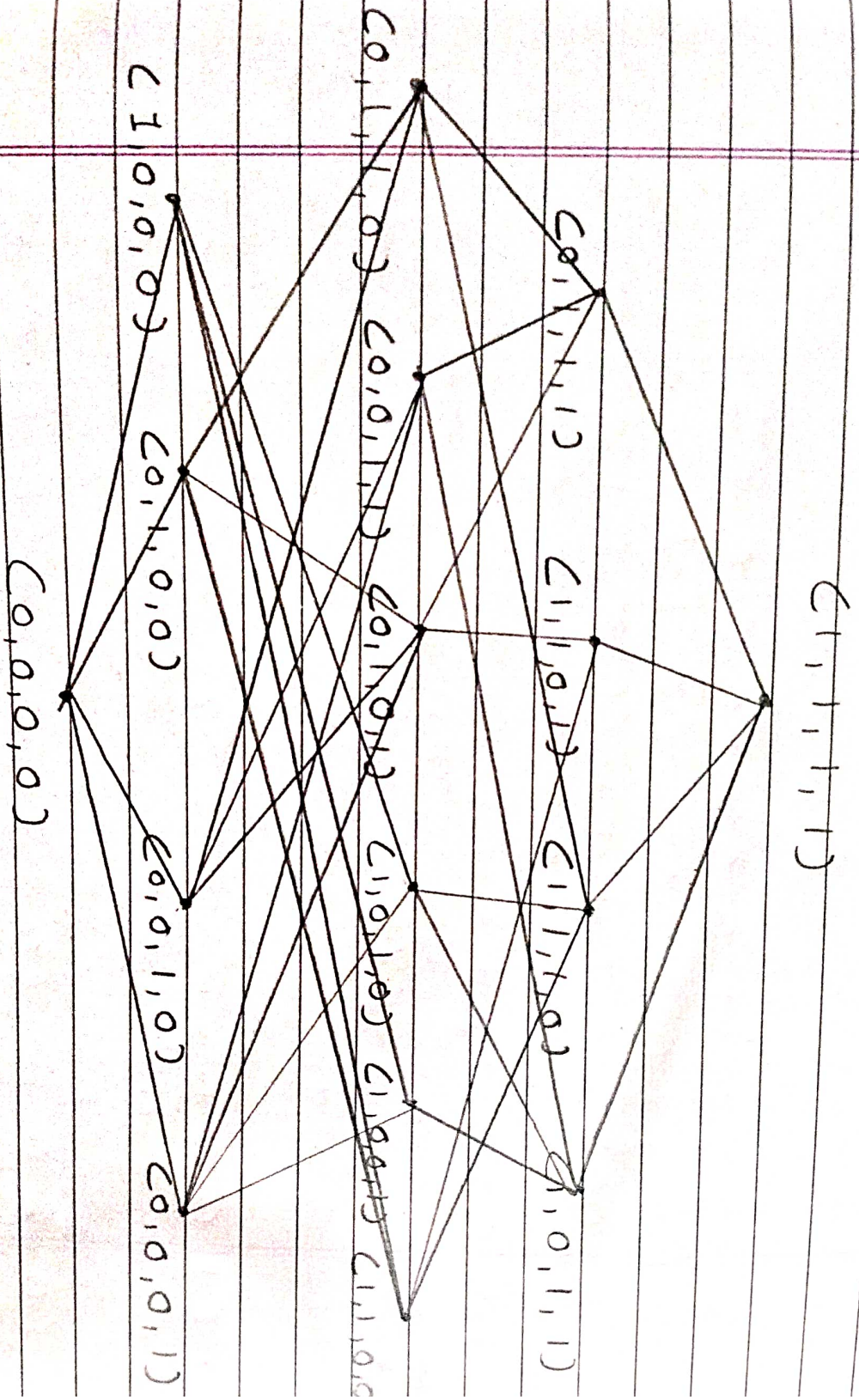
$$(1, 1, 0, 1) \rightarrow (1, 1, 1, 1)$$

$$(1, 1, 1, 0) \rightarrow (1, 1, 1, 1)$$

$$(1, 0, 1, 1) \rightarrow (1, 1, 1, 1)$$

$$(1, 1, 1, 1) \rightarrow X$$

$$(0, 0, 0, 1) \rightarrow \{(1, 0, 0, 1), (1, 0, 1, 0), (0, 1, 0, 1)\}$$



* Task: 3

1 Define Properties of Lattice.

Let L be a non-empty set closed under two binary operations called meet and join denoted by \wedge and \vee .

Then L is called a lattice, if the following properties are follow.

(1) Commutative Law:

$$a \wedge b = b \wedge a$$

$$a \vee b = b \vee a$$

(2) Associative Law:

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

$$(a \vee b) \vee c = a \vee (b \vee c)$$

(3) Absorption Law:

$$a \wedge (a \vee b) = a$$

$$a \vee (a \wedge b) = a$$

2 Give an example of poset which is not lattice.

\Rightarrow Consider $\langle \{1, 3, 4, 24, 36, 72\}, \mathcal{D} \rangle$ is Poset.

$$24, 36 \subset \{1, 3, 4, 24, 36, 72\}$$

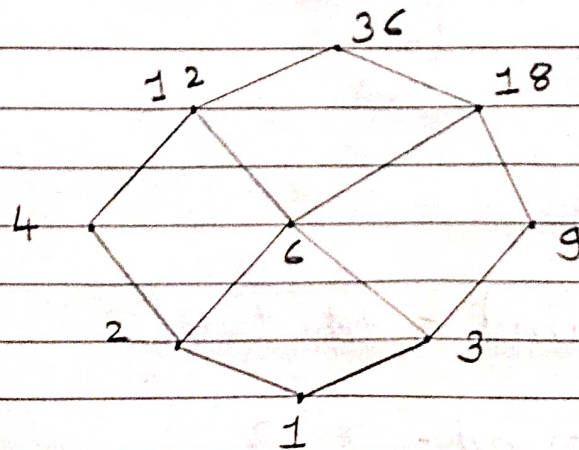
For lattice it follows these three properties.

1 Commutative law:

$$\begin{aligned} 24 * 36 &= \text{GCD} \{24, 36\} \\ &= 12 \notin \{1, 3, 4, 24, 36, 72\} \end{aligned}$$

So, $\langle \{1, 3, 4, 24, 36, 72\}, \mathcal{D} \rangle$ is not lattice.

3 Let $P = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ with divisibility relation. Find GLB and LUB of $\{3, 9\}$ and $\{2, 3\}$.



=> For $\{3, 9\}$

~~GCD~~ $\{3, 9\} = 3$

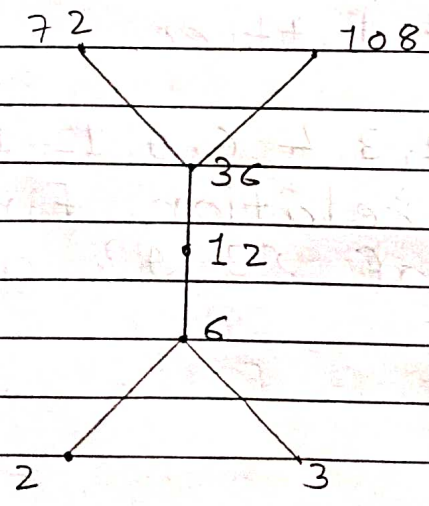
~~LUB~~ $\{3, 9\} = 9$

=> For $\{2, 3\}$

~~GCD~~ $\{2, 3\} = 1$

~~LUB~~ $\{2, 3\} = 6$

4 Let $A = \{1, 2, 3, 6, 12, 36, 72, 108\}$ with divisibility relation. Find upper bound, lower bound, GLB, LUB, maximal and minimal elements.

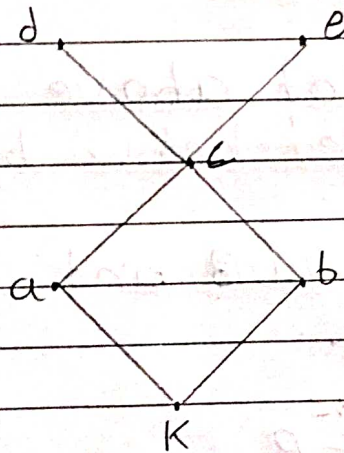


- Upper Bound - 72, 108

- Lower Bound - 2, 3

- Minimal - 2, 3
- Maximal - 72, 108
- ~~G.L.B~~ - G.C.D -
- L.U.B - L.C.M -

5 Let $A = \{K, a, b, c, d, e\}$ be ordered and represent by hasse diagram.
If $A = \{K, a, b\}$ then find upper bound and LUB of A.



6 Give an example of complete lattice which is an infinite lattice.

Let $a, b \in \mathbb{R}$

$L = [a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$ is an infinite lattice.

Let $S \subseteq L, S \neq \emptyset$

$$\begin{aligned} x \in S &\Rightarrow x \in L \\ &\Rightarrow a \leq x \leq b \end{aligned}$$

$\therefore S$ is bounded above by b and S is bounded below by a .

Since, \mathbb{R} has LUB and GLB property.

$$\begin{aligned} &\text{LEM} \\ \therefore \text{LUB}, S \in \mathbb{R} \\ \therefore a \leq \text{LUB} S \leq b \end{aligned}$$

$$\therefore \text{LUB} S \in L$$

$$\begin{aligned} \therefore \text{GLB}, S \in \mathbb{R} \\ \therefore a \leq \text{GLB} S \leq b \\ \therefore \text{GLB} S \in L \end{aligned}$$

$\therefore L = [a, b]$ is an infinite lattice which is complete.

7 Show that $\langle P(X), \subseteq \rangle$ is a bounded lattice where $X = \{a, b, c\}$,

$$X = \{a, b, c\}$$

$$P(X) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\} \}$$

$$\forall \emptyset \subseteq a \subseteq X$$

$$\therefore a \subseteq P(X)$$

$$\therefore \text{G.C.B.}(P(X)) = 1$$

$$\therefore \overset{\text{LCM}}{\text{L.C.B.}}(P(X)) = X$$

\rightarrow 0 element of $\text{G.C.B.}(L) = 0$

\therefore 0 element of $\langle P(X), \subseteq \rangle = \emptyset \in P(X)$

\rightarrow 1 element of $\overset{\text{LCM}}{\text{L.C.B.}}(L) = 1 \in$

\therefore 1 element of $\langle P(X), \subseteq \rangle = X \in P(X)$

Here, 0 and 1 elements are in $P(X)$.

So, $P(X)$ is a bounded lattice.

8 Show that $\langle S_{30}, \mathcal{D} \rangle$ is a bounded lattice.

\rightarrow Here $S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$$1 \mathcal{D} 1 \mathcal{D} 30$$

$$1 \mathcal{D} 2 \mathcal{D} 30$$

$$1 \mathcal{D} 3 \mathcal{D} 30$$

$$1 \mathcal{D} 5 \mathcal{D} 30$$

$$1 \mathcal{D} 6 \mathcal{D} 30$$

$$1 \mathcal{D} 10 \mathcal{D} 30$$

$$1 \mathcal{D} 15 \mathcal{D} 30$$

$$1 \mathcal{D} 30 \mathcal{D} 30$$

$$\therefore 1 \mathcal{D} x \mathcal{D} 30$$

$$\forall x \in S_{30}$$

$$\therefore \text{GCB}(S_{30}) = 1$$

LCM

$$\therefore \text{LUB}(S_{30}) = 30$$

\mathcal{O} -element of $\langle S_{30}, \mathcal{D} \rangle = 1$

\mathcal{I} -element of $\langle S_{30}, \mathcal{D} \rangle = 30$

$\therefore S_{30}$ is a bounded lattice.

* Task : 4

3 Show that $\langle S_{1001}, \mathcal{D} \rangle$ is a lattice.

\Rightarrow We know that $\langle S_n, \mathcal{D} \rangle$ is a Poset $\forall n \in \mathbb{N}$.

$\therefore \langle S_{1001}, \mathcal{D} \rangle$ is a Poset.

$$S_{1001} = \{1, 7, 13, 11, 77, 91, 143, 1001\}$$

$\rightarrow a * b = a \wedge b = \text{GLB} = \text{GCD}$

$a * b$	1	7	11	13	77	91	143	1001
1	1	1	1	1	1	1	1	1
7	1	7	1	1	7	1	1	1
11	1	1	11	1	11	1	13	11
13	1	1	1	13	1	7	11	13
77	1	7	11	1	77	1	11	77
91	1	1	1	17	7	91	13	91
143	1	1	13	11	11	13	143	143
1001	1	7	77	13	77	91	143	1001

$\rightarrow a \vee b = \text{LUB} = \text{LCM}$

$a \vee b$	1	7	11	13	77	91	143	1001
1	1	7	11	13	77	91	143	1001
7	7	7	77	91	77	91	1001	1001
11	11	77	11	143	77	1001	143	1001
13	13	91	143	13	1001	91	143	1001
77	77	77	77	1001	1001	1001	1001	1001
91	91	91	1001	1001	1001	1001	1001	1001
143	143	1001	143	1001	1001	143	143	1001
1001	1001	1001	1001	1001	1001	1001	1001	1001

From the table,

$$a \vee b \in S_{1001}, \quad a \wedge b \in S_{1001}$$

$\therefore \langle S_{1001}, \mathcal{D} \rangle$ is a lattice.

4 Show that $S = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ is a sub lattice of $\langle P(A), \cap, \cup \rangle$ for $A = \{a, b, c\}$

\rightarrow We know that $\langle P(A), \cap, \cup \rangle$ is Poset.

$$A = \{a, b, c\}$$

$$P(A) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \emptyset, \{a, b, c\}\}$$

$$S = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

Let $a, b \in P(A)$

→ For $a \wedge b = GCB$.

\wedge	\emptyset	$\{a\}$	$\{b\}$	$\{a, b\}$
\emptyset	\emptyset	\emptyset	\emptyset	\emptyset
$\{a\}$	\emptyset	$\{a\}$	\emptyset	$\{a\}$
$\{b\}$	\emptyset	\emptyset	$\{b\}$	$\{b\}$
$\{a, b\}$	\emptyset	$\{a\}$	$\{b\}$	$\{a, b\}$

→ For $a \vee b = LUB$ LCM

\vee	\emptyset	$\{a\}$	$\{b\}$	$\{a, b\}$
\emptyset	\emptyset	$\{a\}$	$\{b\}$	$\{a, b\}$
$\{a\}$	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a, b\}$
$\{b\}$	$\{b\}$	$\{a, b\}$	$\{b\}$	$\{a, b\}$
$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$

$\therefore a \vee b \in P(X)$, $a \wedge b \in P(X)$

\therefore So, S is a sub lattice of $\langle P(A), \wedge, \vee \rangle$ where

$$A = \{a, b, c\}$$

5 Show that $S = \{1, 2, 3, 6\}$ is a sub lattice of $\langle S_{30}, \text{GCD}, \text{LCM} \rangle$

$$S_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$$

$$S = \{1, 2, 3, 6\}$$

→ For $a \wedge b = \text{GCD}$

GCD	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

→ For $a \vee b = \text{LCM}$

LCM	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

$$\therefore a \wedge b \in S_{30}, a \vee b \in S_{30}$$

\therefore So, S is a sub lattice of the S_{30} .

1 Find Complement of each element of Lattice $\langle S_{10}, \text{GCD}, \text{LCM}, 1, 10 \rangle$.

2. Find Complement of each element of Lattice $\langle S_{30}, \text{GCD}, \text{LCM}, 1, 30 \rangle$.