Unit : 3 : Propositional Logic

* Task - 1 : Logic.

1 Translate the following propositions into their symbolic form:

(a) Aditya is a musician and Rita is a photographer.

p : Aditya is a Musician
q : Rita is a photographer.

$\therefore$ $p \wedge q$

(b) Krishna will attend the class or will go to picnic.

p : Krishna will attend the class.
q : Krishna will go to picnic.

$\therefore$ $p \vee q$

(c) It is not true that Vivek arya is a doctor and Sushil Gupta is an engineer.

p : Vivek arya is a doctor.

q: Sushil Gupta is an Engineer.

$\therefore N(P \wedge q)$

(d) Discrete Mathematics is interestring or Mathematics is not difficult is general.

P: Discrete Mathematics is interestring.
q: Mathematics is not difficult.

$P \vee \sim q$

(e) $3+4=7$ and $5+8=13$

P: $3+4=7$, q: $5+8=13$

$\cancel{P \vee q}$ $P \wedge q$

2 Let P: Today is Monday.
q: It is raining
r: It is hot.

(a) $P \to q$: If today is Monday then it is raining.

(b) $\sim q \to (r \wedge p)$: If It is not raining then It is hot and today is Monday.

(c) $\sim p \rightarrow (q \vee r)$: If Today is not monday then It is raining or it is hot.

(d) $p \wedge (q \vee r) \rightarrow r \vee (q \vee p)$

If today is monday and It is raining or hot then It is hot ~~raining~~ or It is raining or today is monday.

(e) $\sim (p \vee q) \leftrightarrow r$: It is not true that today is monday or it is raining if and only if it is hot.

3 State Inverse, Converse and Contrapositive of the following implications.

(a) If $5x + 1 = 11$ then $x = 2$

$p : 5x + 1 = 11$, $q : x = 2$

$p \rightarrow q$

(i) Converse : $q \rightarrow p$

If $x = 2$ then $5x + 1 = 11$

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(ii) Inverse : $\sim p \rightarrow \sim q$

~~If $x \neq 2$~~
If $5x + 1 \neq 11$ then $x \neq 2$

(iii) Contrapositive : $\sim q \rightarrow \sim p$

If $x \neq 2$ then $5x + 1 \neq 11$

(b) If you work hard then you can earn money.

p : You work hard
q : You can earn money.

$p \rightarrow q$

(i) Converse : $q \rightarrow p$

If You can earn money then You work hard.

(ii) Inverse : $\sim p \rightarrow \sim q$

~~If You can not earn money then You~~

If You can not work hard then You can not earn money.

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(iii) Contrapositive : $\sim q \rightarrow \sim p$

If You can not kearn money then You can not work hard.

4 Determine the truth value of each of the following propositions, If $3+5>2$ is P and $1+3=4$ is Q.

(a) If $3+5<2$ then $1+3=4$

$\therefore p \rightarrow q$
$\therefore F \rightarrow T = T$

(b) If $3+5<2$ then $1+3 \neq 4$

$\therefore p \rightarrow \sim q$
$\therefore F \rightarrow F = T$

(c) If $3+5>2$ then $1+3 \neq 4$

$\therefore p \rightarrow \sim q$
$\therefore T \rightarrow F = F$

(d) $3+5<2$ if and only if $1+3=4$

$\therefore p \leftrightarrow q$
$\therefore F \leftrightarrow T = F$

(e) $3 + 5 > 2$ if and only if $1+3 = 4$

$\therefore p \leftrightarrow q$

$\therefore T \leftrightarrow T = T$

(f) $3 + 5 < 2$ if and only if $1+3 \neq 4$

$\therefore p \leftrightarrow \sim q$

$\therefore F \leftrightarrow F = T$

5 Represent the given proposition symbollicaily by letting

p : You run 10 laps daily

q : You are healthy

r : You take multivitamins.

(a) If you run 10 laps daily then you will be healthy.

$\therefore p \rightarrow q$

(b) If you do not run 10 laps daily or do not take multivitamins then you will not be healthy.

$\therefore \sim p \vee \sim r \rightarrow \sim q$

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(c) Taking multivitamins is sufficient for being healthy.

$\therefore r \rightarrow q$

(d) You will be healthy if and only if you run 10 laps daily and take multivitamins.

$\therefore q \leftrightarrow (p \wedge r)$

(e) If you are healthy then you run 10 laps daily or you take multivitamins.

$\therefore q \rightarrow (p \vee r)$

(f) If You are healthy and run 10 laps daily then you do not take multivitamins.

$\therefore (q \wedge p) \rightarrow \sim r$

6 Write for the following statements.

(a) If $4 < 6$ then $9 > 12$

(b) $|4| < 3$ if $-3 < 4 < 3$

(i) Each conditional Proposition Symbollically.

(ii) Write the Converse and contrapositive and truth value.

[a] $p : 4 < 6$
$q : 9 > 12$

(i) $\therefore p \rightarrow q$
$\therefore T \rightarrow F = F$

(ii) Converse : $q \rightarrow p : F \rightarrow T : T$

If $9 > 12$ then $4 < 6$

- Contrapositive: $\sim q \rightarrow \sim p : T \rightarrow F = F$

~~If $9 \not> 12$ then $4 \not< 6$~~

If $9 < 12$ then $4 > 6$

[b] $p : |4| < 3$
$q : -3 < 4 < 3$

(i) $q \rightarrow p ; F \rightarrow T = T$

(ii) Converse : $p \rightarrow q : T \rightarrow F = F$

If $|4| < 3$ then $-3 < 4 < 3$

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- Contrapositive : $\sim p \to \sim q$: $T \to T$: T

If $4 \geq |3|$ then $-3 \geq 4 \geq 3$

7 State whether $P \equiv Q$ or not for each proposition P and Q.

(a) $P = p$, $Q = p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Here, $P \neq Q$: So, P and Q are not equivalent.

(b) $P = p \wedge q$, $Q = \sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	F	T

Here, $p \wedge q \neq (\sim p \vee \sim q)$

(c) $P = p \to q$, $Q = -p \vee q$

p	q	$\sim p$	$p \to q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Here $p \to q = (\sim p \vee q)$

(d) $P = p \wedge (\sim q \vee r)$, $Q = p \vee (q \wedge \sim r)$

p	q	r	$\sim q$	$\sim r$	$\sim q \vee r$	$q \wedge \sim r$	$p \wedge (\sim q \vee r)$	$p \vee (q \wedge \sim r)$
T	T	T	F	F	T	F	T	T
T	T	F	F	T	F	T	F	T
T	F	T	T	F	T	F	T	T
T	F	F	T	T	T	F	T	T
F	T	T	F	F	T	F	F	F
F	T	F	F	T	F	T	F	T
F	F	T	T	F	T	F	F	F
F	F	F	T	T	T	F	F	F

Here $p \wedge (\sim q \vee r) \neq p \vee (q \vee \sim r)$

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(e) $P = p \to q$, $Q = \sim q \to \sim p$

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Here, $p \to q \neq (\sim q \to \sim p)$

8 Show that the binary operation disjunction over the set of statements is commutative. Also show by truth table that the statement $(p \vee q) \leftrightarrow (q \vee p)$ is Tautology.

p	q	$p \vee q$	$q \vee p$	$(p \vee q) \leftrightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

From the table, $(p \vee q) \leftrightarrow (q \vee p)$ is commutative.

$(p \vee q) \leftrightarrow (q \vee p)$ is a tautology as it is true for all the values.

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9 Prove that the statement $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

p	q	$p \vee q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

From the table,

$(p \vee q) \wedge (\sim p \wedge \sim q)$ is contradiction for all false value.

10 Examine the vaildity of the arguments.

(a) $p \vee q$
$\sim q$
$\therefore p$

p	q	$p \vee q$	$\sim q$	p
~~T~~	~~T~~	~~T~~	~~F~~	
T	F	T	T	T
~~F~~	~~T~~	~~T~~	~~F~~	
~~F~~	~~F~~	~~F~~	~~T~~	

For the vaild argument,

p ∨ q and ∼q must be true and p is must be ~~False~~ true.

-> From the table, p ∨ q and ∼q are true then p is become True.

So, this argument is ~~in~~vaid.

(b) $p \vee q$
$p \to \sim q$
$p \to r$
$\therefore p$

p	q	r	∼q	p ∨ q	p → ∼q	p → r	p
T	T	T	F	T	F	T	
T	T	F	F	T	F	F	
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	
F	T	T	F	T	T	T	F
F	T	F	F	T	T	T	F
F	F	T	T	F	T	T	
F	F	F	T	F	T	T	

From, the table, p ∨ q, p → ∼q and p → r become True at three time.

But when p ∨ q, p → ∼q and p → r become true then one time p also become True.

So, this argument is invaild.

(c) $p \to q$
$\sim p$
$\therefore \sim q$

p	q	$\sim p$	$\sim q$	$p \to q$
T	T	~~F~~	F	~~T~~
T	F	~~F~~	T	~~F~~
F	T	~~T~~	F	~~F~~
F	F	T	T	T

From the table,

When $p \to q$ and $\sim p$ become true then $\sim q$ is also become True.

So, this argument is ~~in~~vaild.

11 Test the validity of the argument.

(1) If the morning is fine, I go for a walk
I do not go for a walk

$\therefore$ The morning is not fine.

p : The morning is Fine.
q : I go for walk.

Ar~~o~~gument - $p \rightarrow q$

$\sim q$

$\therefore \sim p$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T -

From the table,

When $p \rightarrow q$ and $\sim q$ become true then $\sim p$ is also become True.

So, this argument is ~~in~~vaild.

(2) If income tax rates are lowered, income tax collection increaes.
Income tax collection increaes.

$\therefore$ income tax rates are lowered.

p : Income tax rates are lowered.
q : Income tax collection increaes.

Argument - $p \to q$

q

$\therefore p$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

From the table,

$p \to q$ and q are become true at two places.

But at one places P become true.

So, this argument is unvaild.

(3) Hypothesis: If there is gas in the car, then I will go to the store. If I go to the store, then I will get a soda. There is gas in the car.

Condusion: I will get a soda.

~~P Argument:~~

p : There is gas in car.
q : I will go to the store
r : I will get a soda.

Argument:
$$\begin{array}{c} p \to q \\ q \to r \\ \underline{\quad p \quad} \\ \therefore r \end{array}$$

p	q	r	p->q	q->r
T	T	T	T	T -
~~T~~	T	F	~~T~~	~~F~~
~~T~~	F	T	~~F~~	~~T~~
~~T~~	F	F	~~F~~	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	T

From the table,

p->q, q->r and p become true
then p is also become true.

So this argument is vaild.

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* Task : 2 Quantifiers.

1 Check whether the following statement are propositional function and for ~~su~~ each propositional function, give of discourse domain and truth set.

(a) $(2n+1)^2$ is odd integer.

It is a Propositional Function.
$n \in R$ and $T_n \in Z$.

b) $1+3=4$

It is not a Propositional Function.

c) $x \geq x^2$

It is a Propositional function.
and ~~$n \in$~~ $x \in R$, $T_x \in x : 0 \leq x \leq 1$

d) Let x be a real no.

It is not a Propositional function.

2 Let P(n) be the propositional function "n divides 77". Write following propositions in words and tell whether it is truth or false. The Domain of discourse is Z

P(n) = n divides 77
~~Z ∈~~ $n \in Z$.

(a) P(11) - 11 is divides with 77 means True.

(b) P(8) - False

(c) $\forall n P(n)$ - False

(d) $\exists n P(n)$ - True

3 Let P(x) donate the statement "x is doning walk in morning". The domain of discourse is the set of all people. Write each proposition in words.

~~(a) ∀~~ P(x) = x is doing walk in morning
$x \in$ All People

(a) $\forall_x P(x)$ - All People are doing walk in the morning.

(b) $\exists_x P(x)$ - Some People are doing walk in the morning.

(c) $\sim(\exists_x P(x))$ - It is not true that Some People are ~~not~~ doing walk in the morning.

(d) $\exists_x (\sim P(x))$ - Some People are not doing walk in the morning.

(e) $\sim(\forall_x P(x))$ - It is not true that, all People are doing walk in the morning.

4 Let $P(x)$ denote the statement "x spends more than six hours every weekday in class". The domain of discourse is the set of student.

$P(x)$ - x spends more than six hours every weekday in class

x = set of all student.

(a) $\exists x P(x)$ - Some student spends more than six hours every weekday in class.

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(b) $\forall x\, P(x)$ - All the student spends more than six hours every weekday in class.

(c) $\exists x\, \sim P(x)$ - Some student are not spends more than six hours every weekday in class.

(d) $\forall x\, \sim P(x)$ - All the student are not spends more than six hours every weekday in class.

5 Let $P(x)$ denote the statement "x is an accountant" and Let $Q(x)$ denote the statement "x is owns a Porsche". Write each statement symbolically.

~~(a)~~ $P(x)$ = x is an accountant.
$Q(x)$ = x is owns a Porsche.

(a) All accountants own Porsche.

$\forall x\,(P(x) \rightarrow Q(x))$

(b) Some accountants own Porsche.

$\exists x\,(P(x) \rightarrow Q(x))$

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(c) All owners of Porsches are accountants.

$\forall_x (Q(x) \rightarrow P(x))$

(d) Someone who owns a Porsche is an accountants.

$\exists_x (Q(x) \wedge P(x))$

(d) Write the negation of each proposition in (1), (2), (3) and (4) symbollically and in words.

(a) $\forall_x (P(x) \rightarrow \sim Q(x))$ - All accountants do not own porsche.

(b) $\exists_x (P(x) \rightarrow \sim Q(x))$ - Some accountant do not own porsche.

(c) $\forall_x (Q(x) \rightarrow \sim P(x))$ - No accountant own a porsche.

(d) $\exists_x (Q(x) \wedge \sim P(x))$ - Someone who owns a porsche is not an accountant.

6 Let $P(x)$ denote the statement "x can speak Punjabi" and Let $Q(x)$ denote the statement "x knows the computer languag c++". Exprees each of the following statement in terms of $P(x)$ and $Q(x)$. quantifiers and logical connectives. For the universe of discoure for quantifiers use the set of all student at your school.

$P(x)$ = x can speak Punjabi.
$Q(x)$ = x knows the computer Language.

x = set of all student.

(a) There is a student at your school who can speak Punjabi and who knows c++.

$$\exists_x (P(x) \wedge Q(x))$$

(b) There is a student at your school who can speak Punjabi but who not knows c++.

$$\exists_x (P(x) \wedge \sim Q(x))$$

(c) Every student at your school either can speak Punjabi or knows c++.

$\forall x \, (P(x) \vee Q(x))$

(d) No students at your school can speak Punjabi or knows c++.

$\forall x \sim (P(x) \vee Q(x))$

7 Let $P(x)$, $Q(x)$ and $R(x)$ be the statements "x is Proffesor", "x is ignorent" and "x is vain". Express each of the following statement in terms of $P(x)$, $Q(x)$ and $R(x)$. For the universe of discourse is set of all people.

$P(x)$ = x is a Proffesor.
$Q(x)$ = x is ignorent
$R(x)$ = x is vain

x = set of all people.

(a) No Professors are ignorent.

$\forall x \, (P(x) \rightarrow \sim Q(x))$

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(b) All Ignorent people are vain.

$\forall_x (Q(x) \rightarrow R(x))$

(c) No Professors are vain.

$\forall_x (P(x) \rightarrow \sim R(x))$

8 Let $P(x)$, $Q(x)$, $R(x)$ and $S(x)$ be the statements "x is baby", "x is Logical", "x is able to mange a crocodile" and "x is despised". For the universe of discourse for quantifiers use the set of all people.

$P(x)$ = x is Baby
$Q(x)$ = x is Logical
$R(x)$ = x is able to mange a crocodile.

(a) Babies are illogical.

$\forall_x (P(x) \rightarrow \sim Q(x))$

(b) Nobody is despised who can mange a crocodile.

$\forall_x (R(x) \rightarrow \sim S(x))$

(c) logical persons are despised.

$\forall_x (\sim Q(x) \to S(x))$

(d) Babies cannot manage a crocodile.

$\forall_x (P(x) \to \sim R(x))$

(e) Does 4^{th} statement follows from 1^{st}, 2^{nd} and 3^{rd} statement.

Yes.

9 Give an argument using rules of inference to show that the conclusions follows from the hypothesis:

(a) Hypothesis: Everyone in the class has a graphic calculator. Everyone who has a graphic calculator understands the trigonometric functions.

Conclusion: Raphie who is in the class understands the trigonometric functions.

$P(x)$ = x has a graphic calculator.
$Q(x)$ = x understends than
trigonometrix function.

Let R stands for Raphine.

$$\frac{\begin{array}{c} P(x) \rightarrow q(x) \\ \therefore P(x) \end{array}}{q(R)} \rightarrow \frac{\begin{array}{c} P \rightarrow q \\ P \end{array}}{\therefore q}$$

P	q	$P \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Here, Critical Row has all the true value.

So, this argument is vaild by Modus Ponens.

(b) For every real number x, if x is an integer then x is a rational number. The number $\sqrt{2}$ is not rational. Therefore $\sqrt{2}$ is not an integer.

$P(x)$ = x is an integer

$Q(x)$: x is a rational number.

Let x stands for $\sqrt{2}$.

$$\therefore \begin{array}{l} P(x) \to q(x) \\ \underline{\quad \sim q(x)} \\ \therefore \sim P(x) \end{array} \quad \to \quad \begin{array}{l} p \to q \\ \underline{\quad \sim q} \\ \quad \sim p \end{array}$$

$p \to q$	$\sim q$	$\sim p$	p	q
T	F	F	T	T
F	T	F	T	F
T	F	T	F	T
T	T	T	F	F

Here, Critical Row has all the true value.

So, This argument is vaild by Modus Tollends.

(c) Everyone loves Microsoft or Apple.
Lynn does not love Microsoft.
Show that the conclusion Lynn loves Apple.

$P(x)$: x loves Microsoft
$Q(x)$: x loves Apple.

x stands for Lynn.

$$\begin{array}{l} P(x) \vee q(x) \\ \underline{\quad \sim P(x)} \\ \therefore q(x) \end{array} \rightarrow \begin{array}{c} p \vee q \\ \underline{\sim p} \\ q \end{array}$$

p	q	$p \vee q$	$\sim p$	~~q~~
T	T	T	F	
T	F	T	F	
F	T	T	T	
F	F	F	T	

Here, Critical row has all the true value.

So this argument is vaild by Disjunctive syllogism.

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Task: 3 Proof Techniques.

Prove that the Product of two odd integers is an odd integer. Give the name of method of Proof.

Let a and b are two odd integrs.

Suppose: $a = 2m + 1$
$b = 2n + 1$
where $m, n \in Z$

We have to do two odd number Product.

So, $ab = (2m+1)(2n+1)$

$= 4mn + 2m + 2n + 1$

$= 2(mn + m + n) + 1$

$= 2P + 1$
where $P = mn + m + n$

Here, We get odd number.
So, Hence Prove that the Product of two odd integers is an odd integer.

This is prove by direct Proof method.

2 Prove that between two rational numbers, there is always a rational number.

Let a and b be rational numbers. Since the set of rationals is closed under the addition and division,

So, between the a and b rational number is $\frac{a+b}{2}$ 3

Assume that, $a < b$ Then,

→ First we add the a both side.

$\therefore a + a < b + a$

$\therefore 2a < a + b$

$\therefore a < \frac{a+b}{2}$ — (1)

→ Similarlly, we add the b both side

$\therefore a + b < b + b$

$\therefore \frac{a+b}{2} < b$ — (2)

By eq^n 1 and 2,

$$a < \frac{a+b}{2} < b.$$

Thus between two rational numbers, there is always a rational number. and prove method is direct proof.

3 Let n be an integer. Prove if n^2 is odd, then n is odd.

Here, n is a odd number.

Suppose $n = 2k+1$ for some integer k.

then $n^2 = (2k+1)^2$

$$= 4k^2 + 4k + 1$$

$$n^2 = 2(2k^2 + 2k) + 1$$

Here n^2 number is also prove odd.

So, If n^2 is odd then n is odd.

Hence, Prove if n^2 is odd then n is odd.

4 Prove that there is no rational number P/q whose square is 2, that is show that $\sqrt{2}$ is an irrational number.

→ Assume that a rational number $x = p/q$ is equal to 2

Let take x squere.

$$\therefore \frac{p^2}{q^2} = 2$$

$$\therefore p^2 = 2q^2 \quad \text{— (1)}$$

p^2 is an even integeger and it can expressed in the form of 2k.

$$\therefore 2k = 2q^2$$

$$\therefore q^2 = k$$

The square of an odd integer is always odd. so P cannot be an odd.

So, P can be expressed in the form 2k.

$\therefore \; P = 2k$

By $eq^n - 1$

$\therefore \; (2k)^2 = 2q^2$

$\therefore \; 4k^2 = 2q^2$

$\therefore \; q^2 = 2k^2$

q^2 is an even integer.

So, P and q are both the even integer.

This contradiction means that $x = P/q$ whose square is not equal to 2.

5 Prove or disprove that $P(n) = n^2 - n + 41$ is prime for all $n \in N$.

For Prime number, $n \in (m+1)$ is a prime number.

Let $n \in k+1$ and It is prime.

$\therefore \; P(k+1) = (k+1)^2 - (k+1) + 41$

$\therefore \; P(k+1) = k^2 + k + 41$

$P(K+1) = K^2 + K + 41$

which is true for $n = K+1$

$\therefore P(n) = n^2 - n + 41$ is prime number.