Unit: 2 Vectors in R^n

* Task : 1 Vector in R^n

(i) Let $x = (2, 3, 4)$, $Y = (3, 0, 5)$,
$U = (2, 3, 4, 5)$, $V = (4, 6, 9, 2)$

a $x + Y$

$$x + Y = (2, 3, 4) + (3, 0, 5)$$
$$= (5, 3, 9)$$

b $3V + 4V$

$$3V + 4V = 3(4, 6, 9, 2) + 4(4, 6, 9, 2)$$
$$= (12, 18, 27, 6) + (16, \overset{2}{6}4, 36, 8)$$
$$= (28, 82, \underset{51}{63}, 14)$$

c $3U - 4V$

$$3U - 4V = (6, 9, 15, 15) - (16, \overset{2}{6}4, 36, 8)$$
$$= (-10, -15, -21, 7)$$

Page No.
Date :

c ii) Let $U = (4, 1, 2, 3)$, $V = (0, 3, 8, -2)$
and $W = (3, 1, 2, 2)$

a $\|U\| + \|V\|$

$\|U\| = \sqrt{16 + 1 + 4 + 9} = \sqrt{30}$

$\|V\| = \sqrt{0 + 9 + 64 + 4} = \sqrt{77}$

$\|U\| + \|V\| = \sqrt{77} + \sqrt{30}$

b $\|3U - 5V + W\|$

$3U = (12, 3, 6, 9)$

$5V = (0, 15, 40, -10)$

$3U - 5V + W = (12, 3, 6, 9) - (0, 15, 40, -10) + (3, 1, 2, 2)$

$= (15, -11, -32, 21)$

$\|3U - 5V + W\| = \sqrt{225 + 121 + 1024 + 441}$

$= \sqrt{1811}$

c $\left\| \frac{1}{\|W\|} W \right\|$

$$\|w\| = \sqrt{9+1+4+4} = \sqrt{18}$$

$$\frac{w}{\|w\|} = \left(\frac{3}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{2}{\sqrt{18}}, \frac{2}{\sqrt{18}}\right)$$

$$\left\|\frac{w}{\|w\|}\right\| = \frac{\sqrt{18}}{\sqrt{18}} = 1$$

d $\|u+v\|$

$$u+v = (4, 4, 10, 1)$$

$$\|u+v\| = \sqrt{16+16+100+1} = \sqrt{133}$$

(iii) Find the distance between the following vectors.

a $u = (0, 2, 3)$, $v = (1, 2, -4)$

$$\bar{u} - \bar{v} = (-1, 0, 7)$$

$$\|\bar{u} - \bar{v}\| = \sqrt{1+49} = \sqrt{50}$$

b $u = (3, 4, 0, 1)$, $v = (2, 2, 1, -1)$

$$\bar{u} - \bar{v} = (1, 2, -1, 2)$$

$$\|\bar{u} - \bar{v}\| = \sqrt{1+4+1+4} = \sqrt{10}$$

Page No.
Date :

(iv) Determine whether the following vectors are orthogonal,

a $U = (-1, 3, 2)$, $V = (4, 2, -1)$

$U \cdot V = -4 + 6 - 2$

$= 0$

Here, $U \cdot V = 0$, therefore U and V are orthogonal vector.

b $U = (4, 2, 6, -8)$, $V = (-2, 3, -1, -1)$

$U \cdot V = -8 + 6 - 6 + 8 = 0$

Here, $U \cdot V = 0$, therefore U and V are orthogonal vector.

c $U = (1, 2, 3, -4)$, $V = (0, -3, 1, 0)$

$U \cdot V = 1 - 6 + 3 - 4$
$= -7 \neq 0$

Here, $U \cdot V \neq 0$

therefor U and V are orthogomal vector.

(V) Find the angle between to vector.

a $U = (2, -1, 3)$, $V = (2, 5, -5)$

$U \cdot V = 4 - 5 - 15$

$= -16$

$\|U\| = \sqrt{4+1+9} = \sqrt{14}$

$\|V\| = \sqrt{4+25+25} = \sqrt{54}$

$\cos\theta = \dfrac{U \cdot V}{\|U\| \cdot \|V\|}$

$= \dfrac{-16}{\sqrt{14} \cdot \sqrt{54}}$

$\cos\theta = -0.58189$

$\theta = \cos^{-1}(-0.58189)$

$\theta = \cos^{-1}\left(\dfrac{-8\sqrt{21}}{63}\right)$

$\theta = 125.58^\circ$

b $U = (2, 6, 3, 3, -1)$ and $V = (-5, 2, 2, -2, 2)$

$U \cdot V = -10 + 12 + 6 - 6 - 2$

$= 0$

$\|U\| = \sqrt{4+36+9+9+1} = \sqrt{59}$

$\|V\| = \sqrt{25+4+4+4+4} = \sqrt{41}$

$\therefore \cos\theta = \dfrac{U \cdot V}{\|U\| \cdot \|V\|}$

$\therefore \cos\theta = 0$

$\therefore \theta = 90^\circ$

ii) Verify cauchy - schwarz inequallity for following vectors.

a $U = (-4, 2, 1)$, $V = (8, -4, -2)$

$|U \cdot V| = |(-4, 2, 1) \cdot (8, -4, -2)|$

$= |-32 - 8 - 2|$

$= 42$ — 1

$\|U\| = \sqrt{16+4+1} = \sqrt{21}$

$\|V\| = \sqrt{64+16+4} = \sqrt{84}$

$\|U\| \cdot \|V\| = \sqrt{21} \cdot \sqrt{84}$

$= 42$ — (2)

From eq^n 1 and 2

$|U \cdot V| = \|U\| \cdot \|V\|$

Hence, Cauchy- schwarz inequallity is Verify.

6 $U = (-3, 1, 0)$, $V = (2, -1, 3)$

$|U \cdot V| = |(-3, 1, 0) \cdot (2, -1, 3)|$

$= |-6 - 1 + 0|$

$|U \cdot V| = 7$ — (1)

$\|U\| = \sqrt{9+1} = \sqrt{10}$

$\|V\| = \sqrt{4+1+9} = \sqrt{14}$

$\|U\| \cdot \|V\| = \sqrt{10} \cdot \sqrt{14} = 11:83$ — (2)

Page No.
Date : / /

$|U \cdot V| < \|U\| \cdot \|V\|$

Hense, Cauchy-schwarzd inequality is verify.

c $U = (0, -5, 6)$, $V = (4, 7, 3)$

$|U \cdot U| = |(0, -5, 6) \cdot (4, 7, 3)|$

$= |0 - 35 + 18|$

$= 17$ — (1)

$\|U\| = \sqrt{0 + 25 + 36} = \sqrt{61}$

$\|V\| = \sqrt{16 + 49 + 9} = \sqrt{74}$

$\|U\| \cdot \|V\| = \sqrt{61} \cdot \sqrt{74}$

$= 67.18$ — (2)

From eq^n 1 and 2

Hence, Cauchy - schwarz inequality is verify.

$|U \cdot V| < \|U\| \cdot \|V\|$

d $U = (0, -2, 2, 1), \quad V = (-1, -1, 1, 1)$

$|U \cdot V| = |(0, -2, 2, 1) \cdot (-1, -1, 1, 1)|$

$= |2 + 2 + 1|$

$= 5 \quad — (1)$

$\|U\| = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$

$\|V\| = \sqrt{1 + 1 + 1 + 1} = \sqrt{4} = 2$

$\|U\| \cdot \|V\| = 3 \times 2 = 6 \quad — (2)$

From eq^n 1 and 2

$|U \cdot V| < \|U\| \cdot \|V\|$

Hence, Cauchy-schwarz inequality is verify.