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Unit : 1 Matrix Theory And Application of Matrices.

* Task : 1 : Definiton of Special Types of Matrix.

1 Write the definiton of following turms.

(i) Scalar Matrix : In this matrix diagonal elements are equal and onther elements are zero.

Ex. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(ii) Unit Matrix : In this matrix diagonal elements are one and onther elements are zero.

Ex. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(iii) Upper Triangular Matrix : In this matrix diagonal upper elements are non-zero and onther elements are zero.

Ex. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

(iv) Lower Trangular Matrix : In this matrix above the diagonal all elements are zero.

Ex. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 5 & 6 & 7 \end{bmatrix}$

(v) Square Matrix : In this matrix number of row and colume are equal.

Ex. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$

(vi) Trace of Matrix : The sum of main diagonal elements.

Ex. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\therefore 1 + 4 = 5 = tr(A)$

(vii) Transpose of Matrix : In this matrix interchanging number of row and colume.

Ex. $A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$

(viii) **Determinant of Matrix**: This is written by det(A).

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \Rightarrow \det(A) = a_{11}a_{22} - a_{12}a_{21}.$$

Ex. $$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow \det(B) = 5 - 6 = 1$$

(ix) **Symmetric Matrix**: In this matrix, matrix A and Transpose A^T is equal.

Ex. $$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$\therefore A = A^T$$

(x) **Skew-Symmetric Matrix**: In this matrix, matrix A and transpose of matrix is opposite.

Ex. $$A = \begin{bmatrix} 0 & 2 \\ -2 & -0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 0 & -2 \\ 2 & -0 \end{bmatrix}$$

$$A^T = -A$$

(xi) **Orthogonal Matrix**: In this matrix,

$$AA^T = A^TA = I$$

thus, $A^{-1} = A^T$

(xii) Hermitian Matrix: In this matrix, matrix A and transposed of conjugate matrix is equal,

$$A^{\theta} = A$$

(xiii) Skew-Hermitian Matrix: In this matrix A and transposed of conjugate matrix is not equal,

$$\therefore A^{\theta} = -A$$

(xiv) Unitary Matrix: In this matrix,

$$AA^{\theta} = A^{\theta} = I, \text{ thus, } A^{-1} = A^{\theta}$$

(xv) Singular Matrix: In this matrix, determinant is zero,

Ex. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow |A| = 0$

(xvi) Inverse Matrix: For any matrix,

$$AB = BA = I$$

Formula $A^{-1} = \dfrac{adjA}{|A|}$

2 Express the following matrix as the sum of symmetric and skew symmetric matrix,

a $A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 4 & 1 & -5 \\ 2 & 3 & 0 \\ -3 & -6 & 7 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 8 & 3 & -8 \\ 3 & 6 & -6 \\ -8 & -6 & 14 \end{bmatrix}$

$A - A^T = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -6 \\ -2 & 6 & 0 \end{bmatrix}$

$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$= \begin{bmatrix} 4 & 3/2 & -4 \\ 3/2 & 2 & -2 \\ -4 & -3 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 1/2 & 1 \\ -1/2 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$

6 $\begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix} = A$

$A^T = \begin{bmatrix} 1 & -1 & 8 \\ 5 & -2 & 2 \\ 7 & -4 & 13 \end{bmatrix}$

$\Rightarrow A + A^T = \begin{bmatrix} 2 & 4 & 15 \\ 4 & -4 & -2 \\ 15 & -2 & 26 \end{bmatrix}$

$\Rightarrow A - A^T = \begin{bmatrix} 0 & 6 & -1 \\ -6 & 0 & -6 \\ 1 & 6 & 0 \end{bmatrix}$

$\Rightarrow A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$= \begin{bmatrix} 1 & 2 & 15/2 \\ 2 & -2 & -1 \\ 15/2 & -1 & 13 \end{bmatrix} + \begin{bmatrix} 0 & 3 & -1/2 \\ -3 & 0 & -3 \\ 1/2 & 3 & 0 \end{bmatrix}$

iii Express the following matrix as the sum of Hermitian and skew Hermitian matrix.

$$A = \begin{bmatrix} 2+3i & 0 & 4i \\ 5 & i & 8 \\ 1-i & -3+i & 6 \end{bmatrix}$$

$$A^{\theta} = \begin{bmatrix} 2-3i & 5 & 1+i \\ 0 & -i & -3-i \\ -4i & 8 & 6 \end{bmatrix}$$

$$A + A^{\theta} = \begin{bmatrix} 4 & 5 & 1+5i \\ 5 & 0 & 5-i \\ 1-5i & -5+i & 12 \end{bmatrix}$$

$$A - A^{\theta} = \begin{bmatrix} 6i & -5 & -1+3i \\ 5 & 2i & 11+i \\ 1+3i & -11+i & 0 \end{bmatrix}$$

$$\Rightarrow A = \frac{1}{2}(A + A^{\theta}) + \frac{1}{2}(A - A^{\theta})$$

$$= \frac{1}{2}\begin{bmatrix} 4 & 5 & 1+5i \\ 5 & 0 & 5-i \\ 1-5i & -5+i & 12 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 6i & -5 & -1+3i \\ 5 & 2i & 11+i \\ 1+3i & -11+i & 0 \end{bmatrix}$$

iv. Express the Hermitian matrix A as ~~A~~ P + iQ where P is a real symmetric and Q is a skew symmetric matrix.

$$A = \begin{bmatrix} 1 & -i & 1+i \\ i & 0 & 2-3i \\ 1-i & 2+3i & 2 \end{bmatrix}$$

$$A^{\theta} = \begin{bmatrix} 1 & -i & 1+i \\ i & 0 & 2-3i \\ 1-i & 2+3i & 2 \end{bmatrix}$$

$$\cancel{A+A^{\theta}} = \begin{bmatrix} \cancel{2} & \cancel{-2i} \end{bmatrix}$$

$$\Rightarrow P = \frac{1}{2}(A + A^{\theta})$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 4 \\ 2 & 4 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\Rightarrow Q = \frac{1}{2}(A - A^{\theta})$$

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$$Q = \frac{1}{2}\begin{bmatrix} 0 & -2i & -2i \\ 2i & 0 & -6i \\ 2 & 6i & 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & -i & -i \\ i & 0 & -3i \\ 1 & 3i & 0 \end{bmatrix}$$

(v) Prove that matrix $A = \begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is unitary and hence Find A^{-1}.

$$\Rightarrow A^{\theta} = (\bar{A})^T = \begin{bmatrix} \sqrt{2} & i\sqrt{2} & 0 \\ -i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$\rightarrow$ We know that, $AA^{\theta} = I$

$$AA^{\theta} = \begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AA^{\theta} = I$$

$$A^{\theta} = I \cdot A^{-1}$$

$$\therefore A^{-1} = \begin{bmatrix} \sqrt{2} & i\sqrt{2} & 0 \\ -i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Thus, Since A is unitary matrix and $A^{-1} = A^{\theta}$

Vi Verify if the following matrices are orthogonal and hence find their inverses.

$$A \quad \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}.$$

$$A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

-> We know that, $AA^T = I$

$$AA^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ +2 & +2 & -1 \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ A is orthogonal matrix,
Thus

$$A^{-1} = A^T$$

$$\therefore A^T = A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & +2 \\ +2 & -2 & -1 \end{bmatrix}$$

* Task : 2 Row - Echelon From and Reduce Row - Echelon From,

(i) Find row echelon from of the following matries.

a $$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 3 & 1 & 2 & -1 \\ 1 & 5 & 1 & 3 & 2 \\ -1 & -2 & 0 & -2 & 1 \end{bmatrix}$$

Here, $R_3 \rightarrow R_3 - R_1$
$R_4 \rightarrow R_4 + R_1$

$$= \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 3 & 1 & 2 & -1 \\ 0 & 3 & 1 & 3 & 1 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$= \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 3 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -2 & 2 \end{bmatrix}$$

$R_4 \rightarrow R_4 + 2R_3$

$$= \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 3 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

$R_4/6$

$$= \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 3 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B \begin{bmatrix} 1 & 3 & -1 & 2 \\ -1 & 1 & 1 & 2 \\ 2 & 2 & 1 & 6 \end{bmatrix}$$

Here, $R_2 \to R_2 + R_1$
$R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 4 & 0 & 4 \\ 0 & -4 & 4 & 2 \end{bmatrix}$$

$R_2/4$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & -4 & 4 & 2 \end{bmatrix}$$

$R_3 \to R_3 + 4R_2$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 4 & 6 \end{bmatrix}$$

$R_3/4$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3/2 \end{bmatrix}$$

$$c \quad \begin{bmatrix} 1 & 1 & -2 & 0 & 2 \\ -1 & 0 & -1 & -3 & 2 \\ 2 & 2 & 1 & 2 & 1 \\ -2 & 1 & 2 & -3 & -1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 - 2R_1$
$R_4 \to R_4 + 2R_1$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & 2 \\ 0 & 1 & -3 & -3 & 4 \\ 0 & -1 & 5 & 2 & -3 \\ 0 & 3 & -2 & -3 & 3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$
$R_4 \to R_4 - 3R_2$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & 2 \\ 0 & 1 & -3 & -3 & 4 \\ 0 & 0 & 2 & -1 & 1 \\ 0 & 0 & +7 & -6 & -9 \end{bmatrix}$$

$R_3/2$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & 2 \\ 0 & 1 & -3 & -3 & 4 \\ 0 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 7 & -6 & -9 \end{bmatrix}$$

$R_4 \to R_4 - 7R_3$

$$= \begin{bmatrix} 1 & 1 & -2 & 0 & 2 \\ 0 & 1 & -3 & -3 & 4 \\ 0 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 0 & -5/2 & -11/2 \end{bmatrix}$$

(ii) Find the reduce row enchelon from of the following matries.

a $\begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 3 & 5 & 4 & 1 & 1 \\ 0 & 1 & 3 & 3 & 0 \end{bmatrix}$

Here, $R_3 \to R_3 - 3R_1$

$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 22 & 4 & 1 & 7 \\ 0 & 1 & 3 & 3 & 0 \end{bmatrix}$

$R_3 \to R_3 - 4R_1$
$R_4 \to R_4 - 3R_2$

$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 22 & 0 & 1 & -17 \\ 0 & 65 & 0 & 3 & -18 \end{bmatrix}$

$R_4 \to R_4 - 3R_3$

$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 22 & 0 & 1 & -17 \\ 0 & -1 & 0 & 0 & -33 \end{bmatrix}$

$R_4 / -33$

$$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 22 & 0 & 1 & -17 \\ 0 & 1/33 & 0 & 0 & 1 \end{bmatrix}$$

Here, $R_3 \to R_3 - 3R_1$

$$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & -22 & 4 & 1 & 7 \\ 0 & 1 & 3 & 3 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$

$$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & -22 & 0 & 1 & -17 \\ 0 & 1 & 3 & 3 & 0 \end{bmatrix}$$

$R_4 \to R_4 - 3R_2$

$$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & -22 & 0 & 1 & -17 \\ 0 & 1 & 0 & 3 & -18 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & -22 & 0 & 1 & -17 \\ 0 & 67 & 0 & 0 & 33 \end{bmatrix}$$

$R_4/33$

$$= \begin{bmatrix} 1 & 9 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & -22 & 0 & 1 & -17 \\ 0 & 67/33 & 0 & 0 & 1 \end{bmatrix}$$

(6)
$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$= \begin{bmatrix} 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

$R_1/3$, $R_3 \to R_3 - 3R_1$

$$= \begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & -2 & 4 & -4 & -2 & 0 \end{bmatrix}$$

$$R_2 \to R_2 + R_3$$

$$= \begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 1 & -2 & 2 & 2 & -5 \\ 0 & -2 & 4 & -4 & -2 & 0 \end{bmatrix}$$

$$R_3 \to R_3 + 2R_1$$

$$= \begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 1 & -2 & 2 & 2 & -5 \\ 0 & 0 & 0 & 0 & 2 & -10 \end{bmatrix}$$

$$R_3/2$$

$$= \begin{bmatrix} 1 & -7/3 & 8/3 & -5/3 & 8/3 & 3 \\ 0 & 1 & -2 & 2 & 2 & -5 \\ 0 & 0 & 0 & 0 & 1 & -5 \end{bmatrix}$$

(c) $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & 2 \\ 10 & 3 & 9 & 4 \\ 16 & 4 & 12 & 1 \end{bmatrix}$

$$R_1/6$$

$$= \begin{bmatrix} 1 & 1/6 & 1/2 & 4/3 \\ 4 & 2 & 6 & 2 \\ 10 & 3 & 9 & 4 \\ 16 & 4 & 12 & 1 \end{bmatrix}$$

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$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 10R_1$
$R_4 \to R_4 - 16R_1$

$$= \begin{bmatrix} 1 & 1/6 & 1/2 & 4/3 \\ 0 & 4/3 & 4 & -10/3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -17 \end{bmatrix}$$

$R_2\left(\frac{3}{4}\right)$

$$= \begin{bmatrix} 1 & 1/6 & 1/2 & 4/3 \\ 0 & 1 & 3 & -5/2 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -17 \end{bmatrix}$$

$R_3\left(\frac{1}{-6}\right)$

$$= \begin{bmatrix} 1 & 1/6 & 1/2 & 4/3 \\ 0 & 1 & 3 & -5/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -17 \end{bmatrix}$$

$R_4\left(\frac{1}{-17}\right)$

$$= \begin{bmatrix} 1 & 1/6 & 1/2 & 4/3 \\ 0 & 1 & 3 & -5/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1(6)$

$R_4 \to R_4 - R_3$

$$= \begin{bmatrix} 1 & 1 & 3 & 12 \\ 0 & 1 & 3 & -5/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 29/2 \\ 0 & 1 & 3 & -5/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 + \frac{5}{2} R_3$

$$= \begin{bmatrix} 1 & 0 & 0 & 29/2 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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* Task : 3 : Rank of the matrix.

(i) Find the rank of the following matrices by reducing them to row echelon forms.

a $\begin{bmatrix} -1 & 2 & 3 & 4 \\ 1 & 0 & 3 & -4 \end{bmatrix}$

$R_1 \leftrightarrow R_2$

$= \begin{bmatrix} 1 & 0 & 3 & -4 \\ -1 & 2 & 3 & 4 \end{bmatrix}$

$R_2 \to R_2 + R_1$

$= \begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 2 & 0 & 0 \end{bmatrix}$

$R_2 / 2$

$= \begin{bmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Rank of the matrix is 2.

6 $\begin{bmatrix} 3 & 3 & -4 \\ 1 & 1 & -1 \\ 2 & 1 & -3 \end{bmatrix}$

$R_1 \to R_1/3$

$= \begin{bmatrix} 1 & 1 & -4/3 \\ 1 & 1 & -1 \\ 2 & 1 & -3 \end{bmatrix}$

$R_2 \to R_2 - R_1$
$R_3 \to R_2 - 2R_1$

$= \begin{bmatrix} 1 & 1 & -4/3 \\ 0 & 0 & 1/3 \\ 0 & -1 & -1/3 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$= \begin{bmatrix} 1 & 1 & -4/3 \\ 0 & -1 & -1/3 \\ 0 & 0 & 1/3 \end{bmatrix}$

$R_2(-1)$

$= \begin{bmatrix} 1 & 1 & -4/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1/3 \end{bmatrix}$

$R_3(3)$

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$$= \begin{bmatrix} 1 & 1 & -4/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank of matrix is 3.

c $$\begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$
$R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2/3, \quad R_3 \left(\frac{1}{-2}\right)$

$$= \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$$R_3 \to R_3 - R_2$$
$$R_4 \to R_4 - R_2$$

$$= \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$R_3\left(\frac{1}{-2}\right)$$

$$= \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$R_4 \to R_4 + 2R_3$$

$$= \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of matrix is 3.

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d $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{bmatrix}$

$R_2(-1)$

$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix}$

$R_3 \to R_3 + R_2$

$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

Rank of matrix is 2.

e $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$R_1 \leftrightarrow R_3$

$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 2 & 2 & 1 \\ 3 & -2 & 0 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$R_3 \to R_3 - 3R_1$

$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 2 & 2 & 1 \\ 0 & 4 & 9 & -7 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$R_2 \to R_2/2$

$\cancel{R_3 \to R_3 + 2R_1}$

$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 4 & 9 & -7 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$R_3 \to R_3 - 4R_2$

$R_4 \to R_4 - R_2$

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$$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 5 & -9 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 5 & -9 \end{bmatrix}$$

$R_4 \to R_4 - 5R_3$

$$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 13/2 \end{bmatrix}$$

$R_4\left(\frac{2}{13}\right)$

$$= \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 1 & 1/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rank of matrix is 4.

$$F \begin{bmatrix} 1 & 2 & -1 & -4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + R_1$

$$= \begin{bmatrix} 1 & 2 & -1 & -4 \\ 0 & 0 & 5 & 13 \\ 0 & 0 & 5 & -11 \end{bmatrix}$$

$R_3 / 5$

$$= \begin{bmatrix} 1 & 2 & -1 & -4 \\ 0 & 0 & 1 & 13/5 \\ 0 & 0 & 5 & -11 \end{bmatrix}$$

$R_3 \to R_3 - 5R_2$

$$= \begin{bmatrix} 1 & 2 & -1 & -4 \\ 0 & 0 & 1 & 13/5 \\ 0 & 0 & 0 & -24 \end{bmatrix}$$

$R_4 / -24$

$$= \begin{bmatrix} 1 & 2 & -1 & -4 \\ 0 & 0 & 1 & 13/5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rank of matrix is 3.

(ii) Find the rank of the following matries by redusing the normal forms.

a $\begin{bmatrix} -1 & 2 & 3 & -4 \\ 1 & 0 & 3 & 4 \end{bmatrix}$

$R_2 \leftrightarrow R_1$

$= \begin{bmatrix} 1 & 0 & 3 & 4 \\ -1 & 2 & 3 & -4 \end{bmatrix}$

$R_2 \to R_2 + R_1$

$= \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 2 & 6 & 0 \end{bmatrix}$

$R_2 (2)$

$= \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & 3 & 0 \end{bmatrix}$

$C_3 / 3$

$= \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

$C_3 \to C_3 - C_1$

$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

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$C_3 \to C_3 - C_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - 4C_1$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Rank of matrix is 2.

b $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

$R_3 \leftrightarrow R_1$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 4 & 3 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$
$R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -15 \\ 0 & -5 & -4 \end{bmatrix}$$

$\frac{R_2}{-5}$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -3 \\ 0 & -5 & -4 \end{bmatrix}$$

$$R_3 \to R_3 + 5R_2$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 11 \end{bmatrix}$$

$$R_3/11$$

$$= \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to R_1 - 2R_2$$

$$= \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to R_1 - 10R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 + 3R_3$$

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$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank of matrix is 3.

$$\begin{bmatrix} 1 & -1 & 2 & -3 \\ -2 & 1 & 0 & 2 \\ 1 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$
$R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & -1 & 4 & -4 \\ 0 & 4 & -2 & 7 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_2(-1)$

$$= \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -2 & 7 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$= \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & -4 & 4 \\ 0 & 4 & -2 & 7 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$R_3 \to R_3 - 4R_2$$
$$R_4 \to R_4 - R_2$$

$$= \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & 14 & 9 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

$$C_3 \to C_3 - C_4$$

$$= \begin{bmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & -8 & 4 \\ 0 & 0 & 5 & 9 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$R_3 \to R_3 - 2R_4$$

$$= \begin{bmatrix} 1 & 0 & -3 & 1 \\ 0 & 1 & -8 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$R_1 \to R_1 + 3R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & -8 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$R_4/2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & -8 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 8R_3$

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 4R_3$

$\cancel{R_2 \to R_2 - 12}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$C_4 \to C_4 - C_3$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$C_4 \to C_4 - 12C_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_4 \to R_4 - R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

d $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

$$R_2 \to R_2 - 2R_1$$
$$R_3 \to R_3 - 3R_1$$

$$\approx \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -1 & -2 \end{bmatrix}$$

$$R_2(-3)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2/3 \\ 0 & -1 & -2 \end{bmatrix}$$

$$C_2 \to C_2 - 2C_1$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2/3 \\ 0 & -1 & -2 \end{bmatrix}$$

$$R_3 \to R_3 + R_2$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2/3 \\ 0 & 0 & -4/3 \end{bmatrix}$$

$$C_3(3)$$

$$= \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & -4 \end{bmatrix}$$

$$R_3(-4)$$

$$= \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \to C_3 - 9R_1$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \to C_3 - 2C_2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank of matrix is 3.

e $$\begin{bmatrix} 1 & 2 & 3 & -1 \\ -1 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3(2)$

$$= \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$$= \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3(-1)$

$R_3 \to R_3 + R_2$

$R_4 \to R_4 - R_2$

$$= \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_3(-1)$

$$= \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$C_4 \to C_4 - C_3$

$$= \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$C_3 \to C_3 - 3C_1$

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$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + 2C_2$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of matrix is 3.

F $\begin{bmatrix} 1 & 2 & -1 & -4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$

$R_3 \to R_3 + R_1$
$R_2 \to R_2 - 2R_1$

$$= \begin{bmatrix} 1 & 2 & -1 & -4 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 5 & -11 \end{bmatrix}$$

$$R_2/5 \; , \; R_3/5$$

$$= \begin{bmatrix} 1 & 2 & -1 & -4 \\ 0 & 0 & 1 & 3/5 \\ 0 & 0 & 1 & -11/5 \end{bmatrix}$$

$$= \quad C_3 \leftrightarrow C_2$$

$$= \begin{bmatrix} 1 & -1 & 2 & -4 \\ 0 & 1 & 0 & 3/5 \\ 0 & 1 & 0 & -11/5 \end{bmatrix}$$

$$R_1 \to R_1 + R_2$$

$$= \begin{bmatrix} 1 & 0 & 2 & -17/5 \\ 0 & 1 & 0 & 3/5 \\ 0 & 1 & 0 & -11/5 \end{bmatrix}$$

$$C_3(5)$$

$$= \begin{bmatrix} 1 & 0 & 2 & -17 \\ 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 11 \end{bmatrix}$$

(iii) Find nonsingular matries P and Q for following matrices A such that PAQ are in normal form.

a $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & -4 \\ 3 & 3 & -6 \end{bmatrix}$

$A = I_3 A I_3$

$\therefore \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & -4 \\ 3 & 3 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1$

$\therefore \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -4 \\ 0 & 3 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2/3$

$\therefore \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4/3 \\ 0 & 3 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1/3 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_3 \to R_3 - 3R_2$

$\therefore \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -4/3 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1/3 & 0 \\ 3 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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$C_3 \to C_3 + 2C_1$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4/3 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1/3 & 0 \\ 3 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 / -2$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1/3 & 0 \\ -3/2 & 1/2 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 + \frac{4}{3} C_1$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1/3 & 0 \\ -3/2 & 1/2 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 10/3 \\ 0 & 1 & 0 \\ 0 & 0 & 7/3 \end{bmatrix}$$

6 $\begin{bmatrix} 2 & 1 & 1 & 3 \\ 1 & 0 & 1 & 2 \\ 3 & 1 & 2 & 5 \end{bmatrix}$

$A = I_3 A I_4$

$$\therefore \begin{bmatrix} 2 & 1 & 1 & 3 \\ 1 & 0 & 1 & 2 \\ 3 & 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & -3 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 / -2$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 + C_2$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_4 \to C_4 - 2C_1$$

$$\therefore \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \to C_3 - C_1$$
$$C_4 \to C_4 + C_2$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_4 \to C_4 - C_3$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$$

$$A = I_3 A I_4$$

$$\therefore \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 - 4R_1 \to R_2 \to \frac{R_2}{3}$$
$$R_3 \to R_3 - 2R_1 \to R_3 \to \frac{R_3}{2}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ 0 & 2 & -3 & 2 \\ 0 & 2 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4/3 & 1/3 & 0 \\ -1 & 0 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cancel{R_2 \to \frac{R_2}{3}}$$

$$R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ 0 & 2 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -4/3 & 1/3 & 0 \\ 1/3 & -1/3 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2/2$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/6 & 0 \\ 1/3 & -1/3 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to R_1 + R_2$$

$$\begin{bmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/6 & 0 \\ -2/3 & 1/6 & 0 \\ 1/3 & -1/3 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$C_3 \to C_3 + \frac{3}{2} C_2$$

$$C_4 \to C_4 - C_2$$

$$\begin{bmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/6 & 0 \\ -2/3 & 1/6 & 0 \\ 1/3 & -1/3 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \to C_3 - \frac{1}{2} C_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/6 & 0 \\ -2/3 & 1/6 & 0 \\ 1/3 & -1/3 & 1/2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 1/2 & 1/6 & 0 \\ -2/3 & 1/6 & 0 \\ 1/3 & -1/3 & 1/2 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 3/2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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* Task: 4 Inverse of the Square matrix by Gauss - Jordan.

$$a \quad \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$$

$$\Rightarrow [A \mid I] \sim \left[\begin{array}{ccc|ccc} 2 & 6 & 6 & 1 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$$

$$R_1/2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$
$$R_3 \to R_3 - 2R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & -1 & 1 \end{array}\right]$$

$$R_3 \to R_3 - R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$R_1 \to R_1 - 3R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 7/2 & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$R_1 \to R_1 - 3R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/2 & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -7/2 & 0 & -3 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

b $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

$$\Rightarrow [A \mid I] \sim \left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 \leftrightarrow R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 2 & 3 & 4 & 1 & 0 & 0 \end{array}\right]$$

$$R_2 \to R_2 - 4R_1 \,, \quad R_3 \to R_3 - 2R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 0 & -5 & -15 & 0 & 1 & -4 \\ 0 & -1 & -4 & 1 & 0 & -2 \end{array}\right]$$

$$R_2/-5$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & -1 & -4 & 1 & 0 & -2 \end{array}\right]$$

$$R_1 \rightarrow R_1 - 2R_2$$
$$R_3 \rightarrow R_3 + R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 2/5 & -3/5 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & 0 & -1 & 1 & -1/5 & -6/5 \end{array}\right]$$

$$R_3(-1)$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 2/5 & -3/5 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & 0 & 1 & -1 & 1/5 & 6/5 \end{array}\right]$$

$$R_2 \rightarrow R_2 - 3R_3$$
$$R_1 \rightarrow R_1 + 2R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 4/5 & 9/5 \\ 0 & 1 & 0 & 3 & -4/5 & -14/5 \\ 0 & 0 & 1 & -1 & 1/5 & 6/5 \end{array}\right]$$

C $\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 \\ 3 & -2 & 1 & 6 \end{bmatrix}$

$$\Rightarrow [A|I] = \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & -2 & 1 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - 3R_1$

$$= \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & -3 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$R_3 \to R_3 - 3R_2$

$R_4 \to R_4 - R_2$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -2 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_3$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 & -2 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

$$\cancel{R_2 \to R_2 + R_3}$$

$$R_3 \to (-1) R_3$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 & 2 & 3 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

$$\cancel{R_2 \to R_2 + R_3}$$

$$R_1 \to R_1 - R_4$$

$$R_2 \to R_2 + R_4$$

$$= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & -5 & -3 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 3 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

d $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$\Rightarrow [A|I] \sim \left[\begin{array}{cccc|cccc} 3 & -2 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & -2 & -3 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

$R_1 \leftrightarrow R_3$

$\sim \left[\begin{array}{cccc|cccc} 1 & -2 & -3 & -2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 1 & 0 & 1 & 0 & 0 \\ 3 & -2 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

$R_3 \to R_3 - 2R_1$
$R_4 \to R_4 - 3R_1$

$\sim \cancel{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & \end{bmatrix}}$

$\sim \left[\begin{array}{cccc|cccc} 1 & -1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & -3 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & -3 & 0 & 0 & 1 \end{array}\right]$

$R_1 \to R_1 + R_2$

$R_3 \to R_3 - 3R_2$

$R_4 \to R_4 - R_2$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -2 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + R_3$

$R_2 \to R_2 + R_3$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 & -2 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3(-1)$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 3 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_4$

$R_2 \to R_2 + R_4$

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$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & -5 & -3 & 1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 3 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 & 1 \end{array}\right]$$

$$\sim [I/B]$$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -1 & 1 & -1 \\ -5 & -3 & 1 & 1 \\ 2 & 3 & 1 & 1 \\ -3 & -1 & 0 & 1 \end{bmatrix}$$

* Task : 5 System of Linear Equations

(i) Solve the following system of equations

A $x - y + 2z = 3$
$x + 2y + 3z = 5$
$3x - 4y - 5z = -13$

$\Rightarrow$ Augmented matrix,

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 1 & 2 & 3 & 5 \\ 3 & -4 & -5 & -13 \end{array}\right]$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - 3R_1$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 3 & 1 & 2 \\ 0 & -1 & -11 & -22 \end{array}\right]$$

$R_2/3$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 1 & 1/3 & 2/3 \\ 0 & -1 & -1 & -22 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 1 & 1/3 & 2/3 \\ 0 & 0 & -32/3 & -64/3 \end{array}\right]$$

$$R_3 \left(\frac{-32}{3}\right)$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 1 & 1/3 & 2/3 \\ 0 & 0 & 1 & 32 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 1 & 1/3 & 2/3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\Rightarrow$ Here, $\boxed{z = 2}$

$\therefore \quad y + \frac{1}{3} z = \frac{2}{3}$

$\therefore \quad y = \frac{2}{3} - \frac{2}{3}$

$\therefore \quad \boxed{y = 0}$

$\rightarrow \quad x - y + 2z = 3$

$\therefore \quad x = 3 - 2(2) + y$

$\therefore \quad \boxed{x = -1}$

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B

$$x + 2y - z = 1$$
$$3x - 2y + 2z = 2$$
$$7x - 2y + 3z = 5$$

$\Rightarrow$ Augmented matrix.

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & -2 & 2 & 2 \\ 7 & -2 & 3 & 5 \end{array}\right]$$

$$R_2 \to R_2 - 3R_1$$
$$R_3 \to R_3 - 7R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -8 & 5 & -1 \\ 0 & \cancel{-8} & 10 & -2 \end{array}\right]$$

-16

$$R_2/-8$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & +5/8 & 1/8 \\ 0 & -16 & 10 & -2 \end{array}\right]$$

$$R_3 \to R_3 + 16R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 5/8 & 1/8 \\ 0 & 0 & 20 & 0 \end{array}\right]$$

$R_3 / 20$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 5/8 & 1/8 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

=) Let $z = t$

$\therefore \quad y + \frac{5}{8}t = \frac{1}{8}$

$\therefore \quad y = \frac{1}{8} - \frac{5}{8}t$

$\therefore \quad x + 2y - t = 1.$

$\therefore \quad x = 1 + t - \frac{1}{4} + \frac{5}{4}t$

c $\quad 3x + 2y + z = 3$

$2x + y + z = 0$

$6x + 2y + 4z = 6$

=) Augmented matrix,

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 4 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 6R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & -1 & 1 & -6 \\ 0 & -4 & 4 & -14 \end{array}\right]$$

$R_2(-1)$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 6 \\ 0 & -4 & 4 & -14 \end{array}\right]$$

$R_3 \to R_3 + 4R_2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 0 & 10 \end{array}\right]$$

$\Rightarrow$ Here, System has no solution.

d) $3x_1 + 2x_3 + 2x_4 = 0$

$-x_1 + 7x_2 + 4x_3 + 9x_4 = 0$

$7x_1 - 7x_2 - 5x_4 = 0$

$$\Rightarrow [A:B] = \left[\begin{array}{cccc|c} 3 & 0 & 2 & 2 & 0 \\ -1 & 7 & 4 & 9 & 0 \\ 7 & -7 & 0 & -5 & 0 \end{array}\right]$$

$$R_1 \to R_1 + 2R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 14 & 10 & 20 & 0 \\ -1 & 7 & 4 & 9 & 0 \\ 7 & -7 & 0 & -5 & 0 \end{array}\right]$$

$$R_2 \to R_2 + R_1$$
$$R_3 \to R_3 - 7R_1$$

$$= \left[\begin{array}{cccc|c} 1 & 14 & 10 & 20 & 0 \\ 0 & 21 & 14 & 29 & 0 \\ 0 & -105 & -70 & -145 & 0 \end{array}\right]$$

$$R_3 \to R_3 + 5R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 14 & 10 & 20 & 0 \\ 0 & 21 & 14 & 29 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Here, $x_3 = s$ and $x_4 = t$

$\rightarrow$ $\therefore 21x_2 + 14x_3 + 29x_4 = 0$

$\therefore 21x_2 + 14s + 29t = 0$

$$\therefore x_2 = \frac{-29t - 14s}{21}$$

$\rightarrow$ $x_1 - 7x_2 - 4x_3 - 9x_4 = 0$

$$\therefore x_1 - 7\left[\frac{-29t - 14s}{21}\right] - 4s - 9t = 0$$

$$\therefore x_1 = \frac{-2t - 2s}{3}$$

e $\frac{-1}{x} + \frac{3}{y} + \frac{4}{z} = 30$

$\frac{3}{x} + \frac{2}{y} - \frac{1}{z} = 9$

$\frac{2}{x} - \frac{1}{y} + \frac{2}{z} = 10$

$\Rightarrow$ Augmented matrix

$$[A|B] = \left[\begin{array}{ccc|c} -1 & 3 & 4 & 30 \\ 3 & 2 & -1 & 9 \\ 2 & -1 & 2 & 10 \end{array}\right]$$

$$R_1 \to R_1(-1)$$

$$= \left[\begin{array}{ccc|c} 1 & -3 & -4 & -30 \\ 3 & 2 & -1 & 9 \\ 2 & -1 & 2 & 20 \end{array}\right]$$

$$\therefore R_2 \to R_2 - 3R_1$$
$$R_3 \to R_3 - 2R_1$$

$$= \left[\begin{array}{ccc|c} 1 & -3 & -4 & -30 \\ 0 & 11 & 11 & 99 \\ 0 & 5 & 10 & 70 \end{array}\right]$$

$$R_2 \to R_2/11$$
$$R_3 \to R_3/5$$

$$= \left[\begin{array}{ccc|c} 1 & -3 & -4 & -30 \\ 0 & 1 & 1 & 9 \\ 0 & 1 & 2 & 14 \end{array}\right]$$

$$R_3 \to R_3 - R_2$$

$$= \left[\begin{array}{ccc|c} 1 & -3 & -4 & -30 \\ 0 & 1 & 1 & 9 \\ 0 & 1 & 2 & 5 \end{array}\right]$$

Here, $\frac{1}{z} = 5 \Rightarrow z = \frac{1}{5}$

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$\rightarrow \frac{1}{y} + \frac{1}{2} = 9 \Rightarrow \frac{1}{y} = 9 - 5 = 4$

$\therefore y = \frac{1}{4}$

$\rightarrow \frac{1}{x} - \frac{3}{y} - \frac{y}{2} = 30 \Rightarrow \frac{1}{x} = 2$

$\therefore x = \frac{1}{2}$

F $x + y + z = 3$

$x + 2y - z = 4$

$x + 3y + 2z = 4$

Augment matrix.

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & -1 & 4 \\ 1 & 3 & 2 & 4 \end{array}\right]$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 1 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 - 2R_1$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & -5 \end{array}\right]$$

$R_3 \to R_3(-1)$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 5 \end{array}\right]$$

$\Rightarrow$ Here, $z = 5$

$\therefore \; y - 2z = 1 \Rightarrow y = 1$

$\therefore \; x + y + z = 3 \Rightarrow x = -3$

(ii)

a $\quad -2y + 3z = 3$
$3x + 6y - 3z = -2$
$6x + 6y + 3z = 5$

$\to$ Augmented matrix,

$$[A|B] = \left[\begin{array}{ccc|c} 0 & -2 & 3 & 3 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$= \left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 3 \\ 0 & -6 & 9 & 9 \end{array}\right]$$

$R_1 \to R_1/3$

$R_3 \to R_3 - 3R_2$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & -2/3 \\ 0 & -2 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 \to (-1/2)$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & -2/3 \\ 0 & 1 & -3/2 & -3/2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\to$ Here, $z = t$

$\therefore\ y - \frac{3}{2}z = -\frac{3}{2}$

$\therefore\ y = -\frac{3}{2} + \frac{3}{2}t$

$\therefore \quad x + 2z = \frac{7}{3}$

$\therefore \quad x = \frac{7}{3} - 2t$

6 $\quad 3x + 2y - z = -15$

$5x + 3y + 2z = 0$

$3x + y + 3z = 11$

$-6x - 4y + 2z = 30$

→ Augmented matrix,

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 2 & -1 & -15 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \\ -6 & -4 & 2 & 30 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 + 2R_1$

$$= \left[\begin{array}{ccc|c} 3 & 2 & -1 & -15 \\ 5 & 3 & 2 & 0 \\ 0 & -1 & 4 & 26 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 \to R_1/3$

$$= \left[\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -5 \\ 5 & 3 & 2 & 0 \\ 0 & -1 & 4 & 26 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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$$R_2 \to R_2 - 5R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -5 \\ 0 & -1/3 & 11/3 & 25 \\ 0 & -1 & 4 & 26 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_2 \to (-3) R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -5 \\ 0 & 1 & -11 & -75 \\ 0 & -1 & 4 & 26 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_3 \to R_3 + R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -5 \\ 0 & 1 & -11 & -75 \\ 0 & 0 & -7 & -49 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_3 \to R_3 / -7$$

$$= \left[\begin{array}{ccc|c} 1 & 2/3 & -1/3 & -5 \\ 0 & 1 & -11 & -75 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$z = 7$

$\therefore\ y - 11z = -75 \Rightarrow y = 2$

$\therefore\ x + \frac{2}{3}y - z = -5 \Rightarrow x = \frac{2}{3}$

Find value of a.

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array}\right]$$

$$R_2 \rightarrow R_2 - 3R_1$$
$$R_3 \rightarrow R_3 - 4R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right]$$

$$R_3 \rightarrow R_3 - R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right]$$

=) For $a = 4$, System has infinite solution.

-) For $a \neq 4$, System has unique solution.

-) For $a = -4$, System has no solution.