Unit : 5 : Interpolation

* Task : 1 : Relation Between Operators.

1 Prove that $(1+\Delta)(1-\nabla) = 1$

$\Rightarrow$ L.H.S. $= (1+\Delta)(1-\nabla)$

We know $1+\Delta = E$ and
$1-\nabla = E^{-1}$

$= E \cdot E^{-1}$

$= 1$

L.H.S. = R.H.S.

2 Prove that $\delta = 2\sinh\left(\frac{hD}{2}\right)$

We know that, $E' = e^{hD}$

$\delta f(x) = f(x+h/2) - f(x-h/2)$

$= E^{1/2} f(x) - E^{-1/2} f(x)$

$= f(x)\left[E^{1/2} - E^{-1/2}\right]$

$$\delta f(x) = f(x)\left(e^{hD/2} - e^{-hD/2}\right)$$

$$\delta = 2\sinh\left(\frac{hD}{2}\right) = 2\left(\frac{e^{x} - e^{-x}}{2}\right)$$

$$\left(\because \sinh x = \frac{e^{x} - e^{-x}}{2}\right)$$

Hence, proved $\delta = 2\sinh\left(\frac{hD}{2}\right)$

3 Prove that $\Delta \log f(x) = \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$

$\Rightarrow$ L.H.S. $= \Delta \log f(x)$

$$= \log f(x+h) - \log f(x)$$

$$= \log\left(\frac{f(x+h)}{f(x)}\right)$$

$$= \log\left((1+\Delta)\frac{f(x)}{f(x)}\right)$$

$$= \log\left(1 + \frac{\Delta f(x)}{f(x)}\right)$$

L.H.S. = R.H.S.

4 Prove that $\Delta\nabla = (\Delta - \nabla)$

$\Rightarrow$ L.H.S. $= \Delta\nabla$

$$= (E - 1)(1 - E^{-1})$$

$$= E - EE^{-1} - 1 + E^{-1}$$

$$= E - 1 - 1 + E^{-1}$$

$$= \Delta - \nabla$$

L.H.S. = R.H.S.

5 Prove that $\Delta = E\nabla = \Delta E$

$\Rightarrow$ We know that,

$$\Delta = 1 - E^{-1}$$

$$\Delta = 1 - \frac{1}{E}$$

$$\therefore \Delta E = E - 1 \quad — (1)$$

$$\therefore \Delta = \frac{E - 1}{E}$$

$$\therefore \Delta E = E - 1 \quad — (1)$$

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$\therefore \nabla E = (1 - E^{-1})E = E - 1$ — ②

$\therefore \Delta = E - 1$ — ③

eq^n 1 = 2 = 3

$\Delta = E\Delta = E\nabla$

6 Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$\Rightarrow$ R.H.S. $= \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$= \frac{\Delta^2 - \nabla^2}{\nabla\Delta}$

$= \frac{(\Delta - \nabla)(\Delta + \nabla)}{(\Delta - \nabla)}$

$= (\Delta + \nabla)$

R.H.S. = L.H.S.

7 Find the missing terms in the table.

(a)

X	0	1	2	3	4
Y	1	3	9	?	81

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x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		2			
1	3		4		
		6		$a-19$	
2	9		$a-15$		$124-4a$
		$a-9$		$105-3a$	
3	a		$90-2a$		
		$81-a$			
4	81				

$\therefore \Delta^4 y = 0$

$\therefore 124 - 4a = 0$

$\therefore a = \frac{124}{4} = 31$

(b)

X	2	2.1	2.2	2.3	2.4	2.5	2.6
Y	0.135	Y_2	0.111	0.1	Y_5	0.08	0.074

Missing Term $Y_2 = a$

$Y_5 = b$

X	Y	ΔY	ΔY^2	ΔY^3	ΔY^4	ΔY^5
2	0.135					
		a-0.135				
2.1	a		0.246-2a			
		0.111-a		-0.368+a		
2.2	0.111		-0.122-a		0.401-2a	
		-0.011		0.033+b-a		
2.3	0.1		b-0.089		0.236-4b+a	
		b-0.1		0.269-3b		
2.4	b		0.18-2b		-0.535+6b	
		0.08-b		-0.266-3b		
2.5	0.08		-0.086-b			
		-0.006				
2.6	0.074					

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* Task : 2 : ~~M~~ Newton Gregory Forward and Backward Interpolation.

1 Find Value of Sin(52°) from table.

$\theta°$	45°	50°	55°	60°
$Sin(\theta°)$	0.7071	0.7660	0.8192	0.8660

=) For Finding the value of Sin(52°) we have to use Newton Forward Interpolation,
So, $x = 52°$

$$\therefore \; p = \frac{x - x_0}{h} = \frac{52 - 45}{5} = \cancel{1.6}$$

$$= 1.4$$

$\theta°$	$Sin\theta°$	Δy	$\Delta^2 y$	$\Delta^3 y$
45°	0.7071			
		0.0589		
50	0.7660		-0.0057	
		0.0532		-0.0007
55	0.8192		-0.0064	
		0.0468		
60	0.8660			

$$Y = Y_0 + P\Delta Y + \frac{P(P-1)}{2!}\Delta^2 Y + \frac{P(P-1)(P-2)}{3!}\Delta^3 Y$$

$$Y = 0.7071 + 1.4(0.0584) + \frac{1.4(1.4-1)}{2!}(-0.0057) + \frac{1.4(1.4-1)(1.4-2)}{3!}(-0.0007)$$

$$Y = 0.7071 + 0.08246 - 0.001596 + 0.0000392$$

$$Y = 0.7880$$

Value of $\sin(52^\circ) = 0.7880$

2 Compute cosh(0.56) Using Newton's forward difference

x	0.5	0.6	0.7	0.8
$\cosh(x)$	1.12762	1.18546	1.25516	1.33743

=> For Finding the value of sin(56°) we have to use Newton's Forward Interpolation,

So, $x = 0.56$ and $x_0 = 0.5$

$$\therefore P = \frac{x - x_0}{h} = \frac{0.56 - 0.5}{0.1} = 0.6$$

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x	$\cosh(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0.5	1.1276			
		0.057839		
			0.011865	
0.6	1.1854			-0.000695
		0.069704		
			0.012562	
0.7	1.2551			
		0.082266		
0.8	1.3374			

$$y = y_0 + p\Delta y + \frac{p(p-1)}{2!}\Delta^2 y +$$

$$\frac{p(p-1)(p-2)}{3!}\Delta^3 y$$

$$y = 1.127626 + 0.0347034 + (-0.0014238) + (-0.0000390)$$

$$y = 1.1608665$$

4 Find value of y for $x = 21$ and $x = 28$ from the following.

X	20	23	26	29
Y	0.3420	0.3907	0.4384	0.4848

X	Y			
20	0.3420			
		0.0487		
23	0.3907		-0.0010	
		0.0477		-0.00030
26	0.4384		-0.0013	
		0.0464		
29	0.4848			

=> For Finding the value of $x = 21$ we have to use Newton's Forward Interpolation.

So, $x = 21$, $x_0 = 20$

$$p = \frac{x_0 - x_0}{h} = \frac{21 - 20}{3} = 0.3333$$

$$y = y_0 + p\Delta y + \frac{p(p-1)}{2!}\Delta^2 y +$$

$$\frac{p(p-1)(p-2)}{3!}\Delta^3 y$$

$y = 0.3420 + 0.01623 + 0.000111$
$- 0.0000185$

$y = 0.3583$

⇒ For Finding the value of $x = 28$, we have to use Newton's Backword Interpoletion.

So, $x = 28$, $x_3 = 29$

$$P = \frac{x - x_3}{h} = \frac{28 - 29}{3} = (-0.3333)$$

$$Y = Y_3 + P\nabla Y + \frac{P(P+1)}{2!} \nabla^2 Y +$$

$$\frac{P(P+1)(P+2)}{3!} \nabla^3 Y$$

$Y = 0.4848 + (0.01546) + 0.000144$

$Y = 0.46950$

5 The area A of a circle of diameter d is given for the following values and calculate the area of a circle of a diameter of 105 unit using Newton's backword formula.

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

=) For Finding the value of 105, We have to use Newton's Backword Interpoletion.

So, $x = 105$, $x_n = 100$

$$p = \frac{x_m - x_n}{h} = \frac{105 - 100}{5} = 1$$

d	A	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

$$y = y_n + p\nabla y + \frac{p(p+1)}{2!}\nabla^2 y +$$

$$\frac{p(p+1)(p+2)}{3!}\nabla^3 y + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y$$

$$y = 7854 + 766 + 40 + 2 + 4$$

$$= 8660$$

3 From the following data, Find the number of person earning weekly wages between 100 and 110 rupess.

Wages	Below-80	80-100	100-120	120-140	140-160
No. of Person	250	120	100	70	50

⇒ For Finding the value of wages between 100 and 110, we have to use Newton's Forword Interpoletion.

So, $x = 110$, $x_0 = 80$

$\therefore \quad P = \frac{110 - 80}{20} = 1.5$

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$
80	250				
		120			
100	370		-20		
		100		-10	
120	470		-30		20
	540	70		10	
140	~~70~~		-20		
	590	50			
160	~~50~~				

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 +$$

$$\frac{p(p-1)(p-2)\Delta^3 y_0}{3!} \cancel{\Delta^3 y_0} +$$

$$\frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y = 250 + (1.5)120 + \frac{1.5(1.5-1)(1.5-2)(-20)}{2!} + \frac{1.5(1.5-1)(1.5-2)\cancel{(1.5-3)}\cancel{(20)}(-10)}{3!} +$$

$$\frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(20)}{4!}$$

$$y = 422.65625$$

Here, 422.65625 weekly wages 110 Rs. and

370 weekly wages below 100 Rs.

So, Between 100 and 110 rupess

weakly wages = 422.65625 - 370

= 52.594

* Task : 3 : Gauss Forward and Backward Interpolation.

1 Using the Gauss's Forward to get F(6.5) for following data.

x	4	5	6	7	8	9
$F(x)$	14.141	12.043	10.223	8.647	7.260	6.045

=> For Finding the value of $x = 6.5$ we have to use Gauss's Forward Method,

So, $x = 6.5$ and $x_0 = 6$

$$P = \frac{x - x_0}{h} = \frac{6.5 - 6}{1} = 0.5$$

x	$F(x)$	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$	$\Delta^5 Y$
4	14.141					
		-2.098				
5	12.043		0.278			
		-1.82		-0.034		
6	10.223		0.244		0.089	
6.5		-1.576		-0.055		-0.012
7	8.647		0.189		0.077	
		-1.387		-0.017		
8	7.260		0.172			
		-1.215				
9	6.045					

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$$Y = Y_0 + P(-1.576) + \frac{P(P-1)}{2!}(0.244)$$

$$+ \frac{P(P-1)(P+1)}{3!}(-0.055) +$$

$$\frac{P(P-1)(P-2)(P+1)}{4!}(0.089) +$$

$$\frac{P(P-1)(P-2)(P+1)(P+2)}{5!}(-0.012)$$

$Y = 9.398$

2 Using Gauss's forward interpolation formula, find the population of the year 1986 for data.

Year	1961	1971	1981	1991	2001	2011
Population	14	17	32	43	60	95

⇒ For finding the value of 1986, we have to use Gauss's forward interpolation.

So, $x = 1986$, $x_0 = 1981$

$$\therefore p = \frac{x - x_0}{h} = \frac{1986-1981}{10} = 0.5$$

Year	Population	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	
1961	14					
		3				
1971	17		12			
		15		-8		
1981	32		4		10	
1986		11		2		0
1991	43		6		10	
		17		12		
2001	60		18			
		35				
2011	95					

$$Y = 32 + 0.5(11) + \frac{(0.5)(0.5-1)}{2!}4$$

$$+ \frac{0.5(0.5-1)(0.5+1)}{3!}2 +$$

$$\frac{0.5(0.5-1)(0.5+1)(0.5-2)}{4!}10$$

$$Y = 37.0683$$

3 Use Gauss's Forward Interpolation Formula to find 3.3 from the following data.

x	1	2	3	4	5
y	15.3	15.1	15	14.5	14

=> For Finding the value of 3.3, We have to use Gauss's Forward Interpolation.

So, $x = 3.3$ and $x_0 = 3$

$$P = \frac{x - x_0}{h} = 3.3 - 3 = 0.3$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	15.3				
		-0.2			
2	15.1		0.1		
		-0.1		-0.5	
3	15		-0.4		0.9
3.3		-0.5		0.4	
4	14.5		0		
		-0.5			
5	14				

$$Y = 15 + 0.3(-0.5) + \frac{(-0.5)(-0.5-1)}{2!}(-0.4)$$

$$+ \frac{(-0.5)(-0.5-1)(-0.5+1)}{3!}(0.4) +$$

$$\frac{(-0.5)(-0.5-1)(-0.5-2)(-0.5+1)}{4!}(0.9)$$

$$Y = 14.89$$

4 Find $Y(2.36)$ from the following table by using Gauss's Backward Formula.

x	1.6	1.8	2	2.2	2.4	2.6
y	4.95	6.05	7.89	9.03	11.02	13.46

$\Rightarrow$ For, Finding the value of 2.36, we have to use Gauss's Backward Formula,

So, $x = 2.36$ and $x_0 = 2.2$

$$h = \frac{x - x_0}{h} = \frac{2.36 - 2.2}{0.2} = 0.8$$

X	Y	∇Y	$\nabla^2 Y$	$\nabla^3 Y$	$\nabla^4 Y$	$\nabla^5 Y$
1.6	4.95					
		1.1				
1.8	6.05		0.24			
		1.34		0.06		
2	7.39		0.3		-0.02	
		1.64		0.04		0.09
2.2	9.03		0.34		0.07	
2.36		1.99		0.11		
2.4	11.02		0.45			
		2.44				
2.6	13.46					

$$Y = 9.03 + 0.8(1.64) + \frac{0.8(0.8+1)}{2!}$$

$$0.34 + \frac{0.8(0.8+1)(0.8-1)}{3!} 0.04 +$$

$$\frac{0.8(0.8+1)(0.8-1)(0.8+2)}{4!} 0.07 +$$

$$\frac{0.8(0.8+1)(0.8-1)(0.8+2)(0.8-2)}{5!} 0.09$$

$$Y = 10.62$$

5 From the following table, Find y when $x = 38$.

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.6

=) For Finding the value of 38 we have to use Gauss's Forward Interpolation.

So, $x = 38$ and $x_0 = 35$

$$p = \frac{x - x_0}{h} = \frac{38-35}{5} = 0.6$$

X	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
30	15.9				
		-1			
35	14.9		0.2		
38		-0.8		-0.2	
					-0.2
40	14.1		0		
		-0.8		0	
45	13.3		0		
		-0.8			
50	12.5				

$$y = 4.9 + p(-0.8) + \frac{p(p-1)}{2!}\,0.2$$

$$+ \frac{p(p-1)(p+1)}{3!}(-0.2)$$

$$y = 14.9 - 0.48 - 0.024 \cancel{+01}$$

$$y = 14.413$$

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* Task : 4 : Stirling's and Bessel's Interpolation.

2 Using Stirling's interpolation Formula, to compute $Y(35)$ from the following data.

x	20	30	40	50
Y	512	439	346	243

=> For finding the value of $Y(35)$ we have to use Stirling's interpolation.

So, $x_0 = 35$ and $x_0 = 30$
So,

$$P = \frac{x_0 - x_0}{h} = \frac{35-30}{10} = 0.5$$

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$
20	512			
		-73		
30	$\boxed{439}$		$\boxed{-20}$	
35		-93		10
40	346		-10	
		-103		
50	243			

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$$Y = Y_0 + P\frac{(-73-93)}{2} + \frac{P^2}{2!}(-20)$$

$$= 439 + 0.5\frac{(-73-93)}{2} + \frac{(0.5)^2}{2!}(-20)$$

$$= 439 - 41.5 - 2.5$$

$$= 395$$

3 Use Bessel's interpolation formula to find Y(25) from the following data.

x	20	24	28	32
y	2854	3162	3544	3992

For Finding the value of Y(25) we have to use Bessel's Interpolation.

So, $x = 25$, $x_0 = 24$

$$\therefore P = \frac{x - x_0}{h} = \frac{25-24}{4}$$

$$P = 0.25$$

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x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	2854			
		308		
24	3162		74	
25		$\boxed{382}$		$\boxed{-8}$
28	3544		66	
		448		
32	3992			

$$y = \frac{1}{2}(y_0 + y_1) + \frac{1}{2}(2p-1)\Delta y_0 +$$

$$\frac{p(p-1)}{2!}\frac{(\Delta^2 y_{-1} + \Delta^2 y_0)}{2} +$$

$$\frac{(2p-1)}{2}\left(\frac{p(p-1)}{3!}\right)\Delta^3 y_{-1}$$

$$y = 3353 - 95.5 - 6.5625 - 0.0625$$

$$= 3250.875$$

4 Find $y(3.75)$ from the following table by using Bessel's interpolation formula.

X	2.5	3	3.5	4	4.5	5
Y	24.145	22.043	20.225	18.644	17.262	16.047

⇒ For Finding the value of $Y(3.75)$ we have to use Bessel's interpolation method.

So, $x = 3.75$, $x_0 = 3.5$

$$p = \frac{x - x_0}{h} = \frac{3.75 - 3.5}{0.5} = 0.5$$

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$	$\Delta^5 Y$
2.5	14.145					
		-2.102				
3	22.043		0.284			
		-1.818		-0.047		
3.5	20.225		0.237		0.009	
3.75		-1.581		-0.038		-0.003
4	18.644		0.199		0.006	
		-1.382		-0.032		
4.5	17.262		0.167			
		-1.215				
5	16.047					

$$y = \frac{(18.644 + 20.225)}{2} + \frac{1}{2}(2P-1) \cdot$$

$$(-1.581) + \frac{P(P-1)}{2!}(0.237 + 0.193)$$

$$+ \frac{(2P-1)(P-1)P}{12}(-0.038) +$$

$$\frac{(P+1)P(P-1)(P-2)}{4!} \frac{(0.009 + 0.006)}{2}$$

$$+ \frac{(2P-1)(P-1)(P-2)(P+1)P}{5!}(-0.0003)$$

$$Y = 19.4345 + 0 + (-0.0545) + 0$$

$$+ 0.0000585$$

$$Y = 19.38$$

1 Using the Stirling's interpolation formula to get tan(16°) for following data.

x	0°	5°	10°	15°	20°	25°	30°
$\tan x$	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

=> For Finding the value of tan(16) we have to use, Stirling interpolation formula.

So, $x = 16$, $x_0 = 15$

$$h = \frac{x - x_0}{h} = \frac{16 - 15}{5} = 0.2$$

x	$\tan x$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	0.~~0875~~						
		0.0875					
5	0.0875		-0.0041				
		0.0916		0.0069			
10	0.1763		-0.0028		-0.0032		
		0.0888		0.0101		-0.008	
15 x_0	0.2679		0.0073		-0.0112		0.0229
		0.0961		-0.0011		0.0149	
20	0.3640		0.0062		0.0037		
		0.1023		0.0026			
25	0.4663		0.0088				
		0.1111					
30	0.5774						

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$$y = 0.2679 + \frac{0.2(0.0888+0.0961)}{2}$$

$$+ \frac{(0.2)^2}{2!}(0.0073) + \frac{0.2(0.2^2-1)}{3!}$$

$$\frac{(0.0101+0.0011)}{2} + \frac{0.2^2(0.2^2-1)}{4!}(-0.0012$$

$$+ \frac{(0.2)(0.2^2-1)(0.2^2-4)}{5!}(-0.008-0.0149)$$

$$+ \frac{(0.2)^2(0.2^2-1)(0.2^2-2^2)}{6!}(0.0229)$$

$y = 0.2679 + 0.01849 + 0.00014 -$

$0.000144 + 0.00001792 +$

$0.00002185 + 0.000004836$

$y = 0.28643$

* Task : 5 : Langrange's and Divided Difference Interpolation.

1 Determine the interpolating polynomial of degree three using Langrange's interpolation for the following Table.

X	-1	0	1	3
y	2	1	0	-1

=> Here, we want to evaluate interpolation using Langrange's interpolation,

So, $x_0 = -1$, $x_1 = 0$, $x_2 = 1$, $x_3 = 3$
$y_0 = 2$, $y_1 = 1$, $y_2 = 0$, $y_3 = -1$

By Langrange's Interpolation,

$$y = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 +$$

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$$\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0$$

$$y = \frac{(x+1)(x-0)(x-1)}{(3+1)(3-0)(3-1)}(-1) +$$

$$\frac{(x+1)(x-0)(x-3)}{(1+1)(1-0)(1-3)}(0) +$$

$$\frac{(x+1)(x-1)(x-3)}{(0+1)(0-1)(0-3)}(1) +$$

$$\frac{(x-0)(x-1)(x-3)}{(-1-0)(-1-1)(-1-3)}(2)$$

$$y = \frac{x(x^2-4x+3)}{-4} + \frac{(x^2-1)(x-3)}{3}$$

$$+ \frac{x(x^2-1)}{24}$$

$$y = \frac{1}{24}[-6(x^3-4x^2+3x) + x^3 - x + 8(x^3-3x^2-x+3)]$$

$$y = \frac{1}{24}[x^3-25x+24]$$

2 Use Langrange's formula to fit a polynomial to the data given below.

X	-1	0	2	3
Y	8	3	1	12

=> Here, we want to evaluate interpolation using Langrange's interpolation,

So, $x_0 = -1$, $x_1 = 0$, $x_2 = 2$, $x_3 = 3$
$y_0 = 8$, $y_1 = 3$, $y_2 = 1$, $y_3 = 12$

By Langrange's interpolation,

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

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$$Y = \frac{(x-0)(x-2)(x-3)}{(8-0)(8-2)(8-3)}(8) +$$

$$\frac{(x+1)(x-0)(x-3)}{(1+1)(1-0)(1-3)} 1 +$$

$$\frac{(x+1)(x-2)(x-3)}{(3+1)(3-2)(3-3)} 3 +$$

$$\frac{(x+1)(x-0)(x-2)}{(12+1)(12-0)(12-2)} 12$$

$$Y = \frac{x(x-2)(x-3)}{80} + \frac{x(x+1)(x-3)}{(-4)}$$

$$+ \frac{3(x+1)(x-2)(x-3)}{0} +$$

$$\frac{x(x+1)(x-2)}{130}$$

$$Y = \frac{1}{6}(4x^3 + 4x^2 - 30x + 18)$$

4 Compute F(8) from the following table by using Newton's Divided difference formula.

x	4	5	7	10	11	13
Y	48	100	294	900	1210	2028

=> Here, we want to find value of F(8), for that we have to use Newton's Divided difference Method,

X	Y	$\Delta F(x)$	$\Delta^2 F(x)$	$\Delta^3 F(x)$	$\Delta^4 F(x)$	$\Delta^5 F(x)$
4	48					
		52				
5	100		15			
		97		1		
7	294		21		0	
		202		1		0.
10	900		27		0	
		310		1		
11	1210		33			
		489				
13	2028					

$$y = f(x_0) + (x-x_0)f(x_0,x_1) +$$
$$(x-x_0)(x-x_1)f(x_0,x_1,x_2) +$$
$$(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)$$
$$f(x_0,x_1,x_2,x_3,x_4)$$

$$y = 48 + (8-4)(52) + (8-4)(8-5)15$$
$$+ (8-4)(8-5)(8-7)1 + 0 + 0$$

$$y = 448$$

5 Using Newton's divided difference interpolation formula compute $f(9.2)$ from the following data.

x	8	9	9.5	11
$f(x)$	2.079	2.197	2.251	2.397

=> Here, we want to find value of $f(9.2)$ for that we have to use Newton's Divided Difference Method,

X	F(x)	ΔF(x)	$\Delta^2 F(x)$	$\Delta^3 f(x)$
8	2.0794			
		0.1178		
9	2.1972		-0.00653	
		0.108		0.00046
9.5	2.2512		-0.00515	
		0.0977		
11	2.3978			

$Y = F(x_0) + (x - x_0) F(x_0, x_1) +$

$(x - x_0)(x - x_1) f(x_0, x_1, x_2) +$

$(x - x_0)(x - x_1)(x - x_2) F(x_0, x_1, x_2, x_3)$

$Y = 2.0794 + (9.2 - 8) 0.1178 +$

$(9.2 - 8)(9.2 - 9)(-0.00653) +$

$(9.2 - 8)(9.2 - 9)(9.2 - 9.5)(0.00046)$

$Y = 2.0794 + 0.14136 - 0.001567$

$- 0.00003312$

$Y = 2.2192$

3 Express the function $\frac{3x^2-12x+12}{(x-1)(x-2)(x-3)}$

as a sum of partial function using Lagrange's formula.

=) Here, Given that

$$Y = \frac{3x^2-12x+12}{(x-1)(x-2)(x-3)}$$

So, $x_0 = 1$, $x_1 = 2$, $x_2 = 3$

By Lagrange's formula,

$$Y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} x_0 +$$

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} x_2 +$$

$$\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} x_1$$

$$Y = \frac{(x-2)(x-3)(2)}{(1-2)(1-3)} + \frac{(x-1)(x-3)(-1)}{(2-1)(2-3)}$$

$$+ \frac{(x-1)(x-2)\,2}{(3-1)(3-2)}$$

$$Y = \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}$$