Unit - 6 - Numerical Integration

* Task - 1 : Trapezoidal Rule

1 State Trapezoidal rule with $n = 10$ and evaluate $\int_0^1 e^x \, dx$.

=) For $n = 10$, Trapezoidal Rule

$$\int_a^b f(x) \cdot dx = \frac{h}{2} [(y_0 + y_9) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8)]$$

=) Let Given $f(x) = e^x$ and $b = 1$ and $a = 0$

$$\text{So, } h = \frac{b-a}{n} = \frac{1-0}{10} = 0.1$$

x	0	0.1	0.2	0.3	0.4	0.5
y	1	1.1052	1.2214	1.3499	1.4912	1.6487

x	0.6	0.7	0.8	0.9	0.1
y	1.8221	2.0137	2.2255	2.4596	2.7183

By Trapezoidal Rule,

$$\int_0^1 e^x \cdot dx = \frac{h}{2}[y_0 + y_{10} + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9)]$$

$$= \frac{0.1}{2}[(1 + 2.7183) + 2(1.1052 + 1.2214 + 1.3499 + 1.4918 + 1.6487 + 1.8221 + 2.0137 + 2.2255 + 2.4596)]$$

$$= 0.05[3.7183 + 2(15.3379)]$$

$$\int_0^1 e^x \cdot dx = 1.7197$$

2 Evaluate $\int_0^1 e^{-x^2} \cdot dx$ using trapezoidal rule with $h = 0.1$

Here, Given $F(x) = e^{-x^2}$, $a = 0$, $b = 1$, $h = 0.1$

So, $n = \frac{b-a}{h} = \frac{1-0}{0.1} = 10$

X	0	0.1	0.2	0.3	0.4	0.5
Y	1	0.99	0.9608	0.9139	0.8521	0.7788

X	0.6	0.7	0.8	0.9	1
Y	0.6977	0.6126	0.5273	0.4449	0.3679

By Trapezoidal Rule.

$$\int_0^1 e^{-x^2} \cdot dx = \frac{0.1}{2}[(1+0.3679)+2(0.99+0.9608+0.9139+0.8521+0.7788+0.6977+0.6126+0.5273+0.4449)]$$

$$= 0.05[1.3679+2(6.7781)]$$

$$\int_0^1 e^{-x^2} \cdot dx = 0.7462$$

3 Evaluate $\int_0^1 \frac{1}{1+x^2} \cdot dx$ Using trapezoidal rule with $h = 0.2$

=) Here Given $F(x) = \frac{1}{1+x^2}$ and

$a = 0$, $b = 1$ and $h = 0.2$

So, $n = \frac{b-a}{h} = \frac{1}{0.2} = 5$

x	0	0.2	0.4	0.6	0.8	~~1.9~~ 1
y	1	0.9615	0.8621	0.7353	0.6097	0.5

By Trapezoidal Rule,

$$\int_0^1 \frac{1}{1+x^2} \cdot dx = \frac{0.2}{2} [(1+0.5) + 2(0.9615 + 0.8621 + 0.7353 + 0.6097)]$$

$$\int_0^1 \frac{1}{1+x^2} \cdot dx = 0.78372$$

4 Use trapezoidal rule to evaluate $\int_0^2 \frac{x}{\sqrt{2+x^2}} \cdot dx$ dividing the internal into Four equal parts.

=) Let Given $F(x) = \int_0^2 \frac{x}{\sqrt{2+x^2}}$,

$a = 0$, $b = 2$ and $n = 4$,

So, $h = \frac{b-a}{n} = \frac{2}{4} = 0.5$

X	0	0.5	1	1.5	2
Y	0	0.3333	0.5774	0.7276	0.8165

By Trapezoidal Rule,

$$\int_0^2 \frac{x}{\sqrt{2+x^2}} \cdot dx = \frac{h}{2}\left[y_0 + y_4 + 2(y_1 + y_2 + y_3)\right]$$

$$= \frac{0.5}{2}\left[0 + 0.8165 + 2(0.3333 + 0.5774 + 0.7276)\right]$$

$$= 1.0233$$

5 Find the area bounded by the curve $Y = f(x)$ and the X-axis from $x = 7.47$ to $x = 7.52$ from the following Table.

X	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

$\Rightarrow$ Here, Given $n = 5$, $a = 7.52$ and $b = 7.47$

So, $h = \frac{b-a}{n} = \frac{7.52 - 7.47}{5} = 0.01$

By Trapezoidal Rule,

$$\int_a^b f(x) \cdot dx = \frac{0.01}{2}\left[1.93 + 2.06 + 2(1.96 + 1.98 + 2.01 + 2.03)\right]$$

$$\int_a^b f(x) \cdot dx = 0.09965$$

* Task : 2 : Simpson's 1/3 rule, Simpson's 3/8 rule and Weddle's rule.

1 Evaluate $\int_0^6 \frac{1}{1+x} \cdot dx$ using Simpson's 1/3 rule by taking $h=1$.

$\Rightarrow$ Here, Given $f(x) = \frac{1}{1+x}$, $a=0$, $b=6$ and $h=1$

So, $n = \frac{b-a}{h} = \frac{6-0}{1} = 6$

X	0	1	2	3	4	5	6
Y	1	0.5	0.333	0.25	0.2	0.167	0.143

By Simpson's 1/3 rule,

$$\int_0^6 \frac{1}{1+x} \cdot dx = \frac{h}{3}\left[(y_0+y_7) + 2(y_2+y_4+y_6) + 4(y_1+y_3+y_5)\right]$$

$$= \frac{1}{3}\left(1+0.143 + 2(0.333+0.2+0.143) + 4(0.5+0.25+0.167)\right]$$

$$\int_0^6 \frac{1}{1+x} \cdot dx = 1.959$$

2. Using Simpson's 3/8 rule evaluate $\int_0^1 \frac{1}{1+x^2} \cdot dx$ by taking

$h = 1/6$

⇒ Here, Given $f(x) = \frac{1}{1+x^2}$, $a = 0$, $b = 1$ and $h = 1/6$

So, $n = \frac{b-a}{h} = \frac{1}{1/6} = 6$

X	0	1/6	1/3	1/2	2/3	5/6	1
Y	1	0.9730	0.9	0.8	0.6923	0.5902	0.5

By Simpson's 3/8 rule,

$$\int_0^1 \frac{1}{1+x^2} \cdot dx = \frac{3h}{8} [Y_0 + Y_6 + 2(Y_3) + 3(Y_1 + Y_2 + Y_4 + Y_5)]$$

$$= \frac{3}{8 \times 6} [(1 + 0.5) + 2(0.8) + 3(0.6923 + 0.9 + 0.9730 + 0.5902)]$$

$$\int_0^1 \frac{1}{1+x^2} \cdot dx = 0.7854$$

3 Using Simpon's 3/8 rule evaluate $\int_0^3 \frac{1}{1+x} \cdot dx$ by taking $n=6$ and hence calculate log2.

=> Here, Given $f(x) = \int_0^3 \frac{1}{1+x}$,

$a = 0$, $b = 3$ and $n = 6$.

So, $h = \frac{b-a}{n} = \frac{3}{6} = 0.5$

x	0	0.5	1	1.5	2	2.5	3
Y	1	0.6666	0.5	0.4	0.333	0.2857	0.25

By Simpon's 3/8 rule,

$$\int_0^1 \frac{1}{1+x} \cdot dx = \frac{3(0.5)}{8}\left[(1+0.25)+2(0.4) + 3(0.6666+0.5+0.3333+0.2857)\right]$$

$$\int_0^1 \frac{1}{1+x} \cdot dx = 1.3887$$

$$\Rightarrow \int_0^3 \frac{1}{1+x} \cdot dx = \left|\log(1+x)\right|_0^3$$

$$= \log 4$$

4 Evaluate $\int_{-2}^{6} (1+x^2)^{3/2} \, dx \, dy$ by Simpon's 1/3 rule.

$\Rightarrow$ Here, Given, $f(x) = (1+x^2)^{3/2}$,
$a = -2$, $b = 6$ and $n = 6$

So, $h = \frac{b-a}{n} = \frac{6+2}{6} = 1.3333$

x	-2	-2/3	2/3	2	10/3
y	11.1803	1.7360	1.7360	11.1803	42.1478

14/3	6
108.7088	225.0622

By Simpon's 1/3 rule,

$$\int_{-2}^{6} (1+x^2)^{3/2} = \frac{h}{3} [(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)]$$

$$\int_{-2}^{6} (1+x^2)^{3/2} = \frac{h}{8}[(11.1803 + 225.0622) + 2(1.7360 + 42.1478) + 4(1.7360 + 11.1803 + 108.7088)]$$

$$= \frac{1.333}{3}[236.2425 + 87.7676 + 480.5004]$$

$$\int_{-2}^{6} (1+x^2)^{3/2} \cdot dx = 360.22$$

6 Evaluate $\int_{0}^{6} \frac{1}{1+x^2} \cdot dx$ using weddle rule with $n=6$.

⇒ Here, Given $f(x) = \frac{1}{1+x^2}$, $a=0$, $b=6$ and $n=6$

$$h = \frac{b-a}{n} = \frac{6}{6} = 1$$

X	0	1	2	3	4	5	6
Y	1	0.5	0.2	0.1	0.0588	0.0385	0.0270

By Weddle Rule,

$$\int_{0}^{6} \frac{1}{1+x^2} \cdot dx = \frac{3h}{10}[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$$

$= \cancel{\frac{3}{10}} (3.\cancel{5} + 5(0.5))$

$$\int_0^6 \frac{1}{1+x^2} \cdot dx = 1.3735$$

7 Evaluate $\int_4^{5.2} \log_e x \cdot dx$ using weddle rule with $n=6$.

$\Rightarrow$ Here, Given $f(x) = \log_e x$, $a = 4$, $b = 5.2$ and $n = 6$.

So, $h = \frac{b-a}{n} = \frac{5.2-4}{6} = 0.2$

X	4	4.2	4.4	4.6	4.8	5	5.2
Y	1.3863	1.4351	1.4816	1.5260	1.5686	1.6094	1.6486

By using Weddle rule

$$\int_4^{5.2} \log_e x \cdot dx = \frac{3h}{10} [y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6]$$

$$\int_{4}^{5.2} \log x \, dx = 1.8278$$

5 Evaluate $\int_{0}^{\pi} \frac{\sin^2 x}{5+4\cos x} dx$ Using Simpon's 3/8 rule.

Here, Given $f(x) = \sin^2 x / 5+4\cos x$
$a = 0$, $b = \pi$, $h = \frac{\pi}{4}$, $n = 4$

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$f(x)$	0	0.0639	0.2	0.2302	0

By Simpon's 3/8 rule

$$\int_{0}^{\pi} \frac{\sin^2 x}{5+4\cos x} dx = \frac{3h}{8}\left[y_0 + y_4 + 2y_3 + 3(y_1 + y_2)\right]$$

$$= \frac{3h}{8}\left[0 + 0.4604 + 0.947917\right]$$

$$= 0.3688$$