Unit - 1 Combinatorial Analysis

* Task : 1 : The Basic Principle of Counting:

1 Write the Basic Principle of Counting. Suppose a lincensce plate contains two distinct English alphabat followed by three digit with the first digit not zero. How many diffierent lincensce plate can be print?

=> Basic Counting Principle:

1 Multiplication Rule:

If there are one event occures at m way and secound event occures at n way than doing both things simultaneonsly then total number of ways "m x n".

2 Addition Rule:

If there are two alternative jobs which can be done in 'm' ways and in 'n' ways respectivily then either of two jobs can be done in "m+n" ways.

=>

26	25	9	10	10

In license Plate,

For First letter, there are 26 alphabat can be arrange.

For secound letter, Repetation can not allowed so, there are 25 alphabat can be arrange.

For third digit, zero can not be a first digit of three digit number so, there are 9 digit can be arranged.

For Fourth and Fiveth digit there are 10 digit can be arranged.

Total ways $= 26 \times 25 \times 9 \times 10 \times 10$

$= 5,85,000$

5, 85, 000 license plate can be Print.

2 A Person wants to buy a car. There are two brands of car avaliable in the markeet and each has 3 variant models and each model comes in five different colour. In how many ways to choose a car to buy ?

=> For First car brands,

There are 3 models and 5 colour available.

So Choose First
Car ways = 3×5 = 15 way.

=> Similarly Secound car brands have 3 model and 5 colour.

So Secound Car
Choose ways = 3×5 = 15 way

Total Car
Choose ways = 15 + 15
= 30

3 Find the number of 4 letter words with or without meaning which can be formed out of the letter ROSE, where letter repitation is not allowed.

There are four letter in ROSE.

4	3	2	1

For First letter, there are 4 letter can be arrange.

For Secound letter, there are only 3 letter can be arrange because repitation is not allowed.

For third letter, there are only 2 letter can be arrange because repitation is not allowed.

For Fourth letter, there are only 1 letter can be arrange.

Total Words = 4 X 3 X 2 X 1
= 24 words.

4 In a village out of the total number of people, 80 Percentage of the people own coconut groves, 65 Percentage of the people own Paddy fields. What is minimum Percentage of people own both?

5 How many three digit numbers are there (repetition is allowed)

(a) Which do not contain 2 at all?

8	9	9

For First Position zero and Two are not allowed so there are 8 ways.

For Secound and Third Position, Two is not allowed so there are 9 ways.

So, total ways $= 8 \times 9 \times 9$
$= 648$

(b) Which contain 2 aleast once?

9	10	10

Total number of three digit number ways $= 9 \times 10 \times 10$
$= 900$

Ways of 2 aleast one $= 900 - 648 = 252$

(c) Which contain 2 atmost once?

We can fixed 2 atmost Three ways.

(1) $\boxed{2}\,\boxed{2}\,\underset{9}{\boxed{\ }}$ - Last digit = 9

(2) $\boxed{2}\,\underset{9}{\boxed{\ }}\,\boxed{2}$ - Second ~~git~~ digit = 9

(3) $\boxed{\ }\,\boxed{2}\,\boxed{2}$ - First digit can not be zero So, there are 8 digit.

Total ways = 9 + 9 + 8
= 26

Three digit number contain Three Two only one time.

So, Total ways = 26 + 1
= 27

6 How many licence plates be made using either two distinct letter and four digits or two digit and 4 distinct letter where all digits and letter are distinct.

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=> For Two letter and 4 digit

26	25	10	9	8	7

Here, Repetation is not allowed.

For First letter, there are 26 letter can be arranged.

For Second letter, there are 25 letter can be arranged.

For digit arrangment,

We can arrang 10, 9, 8, 7 digit respectivly.

Total ways = $26 \times 25 \times 10 \times 9 \times 8 \times 7$
= 3276000

=> For ~~4~~2 digit and ~~Two~~ Four letter.

10	9	26	25	24	23

Here, Repetation is not allowed.

For First and Second ~~of~~ digit we can arrange 10 and 9

ways
For 4
26, 25,
Total

=> Total
plate

7 Count
intege
less th
divisibl
no dig

Numb
and
digit

Numb
So, la

ways respectivly.

For 4 letter, we can arrange 26, 25, 24 and 23 letter respectivly.

Total ways = $10 \times 9 \times 26 \times 25 \times 24 \times 23$
$= 32292000$

=) Total licence plate number = $32292000 +$
3276000
$= 35568000$

7 Count the number of positive integers greater than 7000 and less than 8000 which are divisible by 5, Provided that no digits are repeated.

Number is Greater than 7000 and less than 8000 so, First digit of number is fixed.

Number can divisible by 5 So, last digit can be 5 or 0.

7			
			0 or 5
1	8	7	2

Repitation is not allowed.

So, Third and secound digit can be 7 and 8 respectivly.

Total number = $1 \times 8 \times 7 \times 2$
$= 112$

* Task : 2 : Permutations :

2 In how many different ways can the letter of the word OPTICAL be arranged so that the vowels always come together?

Here, Given word is 'OPTICAL'
There are Three vowel and Four consonants.

There are OIA vowel arranged together.

A	I	O				

We can arrange Four consonants in 4! way and vowel can also arranged 3! way.
Vawel can arranged intereleted in 1 way.

So Total

way $= (4+1)! \cdot 3!$
$= 720$ ways.

3 How many distinct Permutation can be formed from all the letter of each word.

(a) THEM:

Here, Given word letter is 4.
So, we can arranged them word in 4! way.

Total way = 4! = 24

(b) Unusual:

Here, Given word letter is 7.
So, we can arranged Unusual word in 7! way.

But, In this word U are repeted in Three time.
So, we have to divide by Three

Total way = $\frac{7!}{3!}$ = 840

(c) sociological

Here, Given word letter is 12.
So, we can arranged sociological word in 12! way.

But, In this word o are repeted in Three time, c are repeted in Two time, i are repeted in Two time and l are repeted in Two time. So, we have to divide the word by 3!, 2!, 2! and 2!.

$$\text{Total way} = \frac{12!}{3! \times 2! \times 2! \times 2!}$$

$$= 9979200$$

5 In how many different ways can the letter of the word 'Mathmatics' be arranged such that the vowels must always come together?

Here, Given word is 'Mathematics'. There are four vowels and 7 consonants.

There are aaei word vowel arranged together.

A	A	E	I							

Here consonanuts are arrange in 7! way and vowel can arrange in 4! way.

But vowel can arranged intereleted in 1! way.

Here A are repeted in Two time and M are repeted in Two time and T are repeted in Two time.
So, we have to divided this word into 2!, 2! and 2!

$$\text{Total way} = \frac{(7+1)!}{2! \times 2!} \times \frac{4!}{2!}$$

$$= 120960$$

6 In how many ways can 5 gentlemen and 5 ladies be seated on round table. So, that no two ladies are together seated? 7

There are 5 Gentlemen and 5 ladies on round table but 2 ladies can not seated together.

(i)
(ii)

There are 5 Gentleman on circular table.

For Gentleman
arrange way $= (5-1)!$
$= 4!$

There are ~~5~~ 5 ladies on circular table but Gentleman position are fixed. So, ladies can arrange in linear way.

For ladies
arrange way $= 5!$

Total ways $= 5! \cdot 4!$
$= 2880$

7 In how many ways can a party of 7 Persons arrange themselves

(i) In a Row of 7 chairs?
(ii) Circular Table chairs?

Here, There are 7 Persons are arrange in Party.

(i) In a 7 chair, we have to arrange 7 Person on chair.

Using linear arrangement,

Total way = 7!

= 5040

(ii) In a circular table, we have to arrange 7 Person on chair.

Using Circular Permutations,

Total way = (7-1)!
= 720

8 Find out the number of ways in which 6 rings of diffierent types can be worn in 3 Fingers?

Here, 6 Rings and 3 Fingers are use.

There are 6 Rings to be worn in 3 Fingers, so 3^6 ways to worn rings.

Ways = 3^6 = 729

1 How many different strings can be formed together using the letter of the word "EQUATION" So that

(a) The vowels always together?
(b) Vowels never come together?

There are 8 word in this letter and There are 5 vowels and 3 consonants in the letter.

(a) Here, e, u, a, i, o are together in a one group.

We have 3 word and 1 Group.

Number of Permutation = $(3+1)! \cdot 5!$
= 2880

(b) In this, vowel are never come together.

Here, we have vowel come together Permutation.

number of Permutation in which vowel are together = 2880.

Here, we have 8 letter.
So, number of Total
Permutation = 8!
= 40,320

Vowel come never
togethe Permutation = 40320 - 2880
= 37440.

4 Find the number of ways of arranging the letter of the word RAMANUJAN so that the relative position of vowels and consonants are not changed.

Here, There are 9 words and In this, There are 5 consonant and 2 vowel in this letter.

Number of consonant arrange
Permutation = 5!

Number of vowel arrange
Permutation = 2!

According to Fundamental Principle of counting,

Permutation = 5! x 2!
= 240

9 25 Buses are running between two places P and Q. In how many ways can a person go from P to Q and return by a different bus?

Here, There are 25 Buses running on two places and There are Two ways for running buses.

Number of Permutation $= {}^{25}P_2$

$= 25 \times 24$

$= 600$ ways.

* Task : 3 : Combinations

1 Define Combination and A commitee of 3 is to be formed from a Group of 20 People. How many diffierent committee are posible?

⇒ Combination:

The number of ways of selecting things out of n different thing is called r combination number of n things it is called combination and it is denoted by nC_r or $\binom{n}{r}$ or $C(n, r)$.

⇒ Here, We have Group of 20 People and we have to select 3 people.

Number of Selection = $20C_3$

= 1140.

There are 1140 ways to select 3 people.

2 A commitee of 3 Persons is so be constituted from a group of 2 men and 3 women.
(a) How many way this be done?
(b) How many way 1 men and 2 women in commitee?

Here, There are 2 men and 3 women but we have to select only 3 Person.

(a)

Men	Women
0	3
1	2
2	1

Number of Selection $= 2C_0 \times 3C_3 + 2C_1 \times 3C_2 + 2C_2 \times 3C_1$

$= 10.$

There are 10 ways to select 3 People.

(b) Here, We have ~~two~~ select 1 men and 2 women.

Number of Selection $= 2C_1 \times 3C_2 = 6$

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There are 6 ways to select 2 women and 1 men.

3 What is the number of ways of choosing the 4 cards from a pack of 52 cards?

In how many of These.

(a) Four card of the same suit
(b) Four Face card
(c) Four card from Four suit
(d) Two card red and Two card black
(e) Same color.

Here, There are 52 cards and we have to select Four card.

Number of Selection $= 52C_4$

$= 270725$

There are 270725 way to select Four card.

(a) There are Four suits. One can be select from $4C_1$ and 4 card from select $13C_4$.

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Number of
Selection $= 13C_4 \times 4C_1$

$= 2860$

There are 2860 way to select same suit.

(b) There are 12 Face cards.

Number of
selection $= 12C_4$

$= 495$

There are 495 ways to select Face card.

(c) There are Four suit. And every suit contain 13 cards.

Number of
Selection $= 13C_1 \times 13C_1 \times 13C_1 \times 13C_1$

$= 28561$

There are 28561 ways to select Four different suit.

(d) There are Two color in cards
Every color contain 26 cards.

Number of
Selection $= 26C_2 \times 26C_2$

$= 105625$

There are 105625 ways to select 2 Red and 2 black color card.

(e) There are Two color in cards
Every color contain 26 cards.

Number of
Selection $= 26C_4 + 26C_4$

$= 29900$

There are 29900 ways to select same color cards.

4 A student is to answer 8 out of 10 questions on an exam.

(a) How many choices has he?
(b) How many if he must answer

the First 3 Question?
(c) How many at least 4 of the First 5 Question?

(a) There are 10 question and we have to select 8 question.

Number of Selection $= 10C_8$

$= 45$

There are 45 ways to select 8 question.

(b) Out of 8 question, 3 question is already selected.
So, we have to select 5 question out of 7 question.

Number of Selection $= 7C_5$

$= 21$

There are 21 ways to select 5 question.

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(c) Here, we have to select atleast 4 question from First 5 question.

Reaming ~~5~~ 3 question we have to select 5 question.

Number of Selection $= 5C_4 \times 5C_5 + 5C_3$

$= 25 + 10$

$= 35.$

There are 35 ways to select question.

5 Out of 5 Mathematicians and 7 Physicists. a comittee select 2 mathematicians and 3 Physicists. In how many ways can be done if

(a) Any Mathematician and any Physicists can be included.
(b) ~~O~~ One Particular Physicist must be on comittee.
(c) Two Mathematicians cannot be on comittee?

(a) There are 5 Mathematicians and 7 Physicists and we have to select 2 Mathematician and 3 Physicists.

Number of Selection $= 5C_2 \times 7C_3$

$= 350$

There are 350 way to select comittee.

(b) Here, one Physicists is already selected. than we have to select 2 Physicists and 2 Mathematician

Number of Selection $= \cancel{4}\,\cancel{7C_2}\ 6C_2 \times 5C_2$

$= 150$

There are 150 way to select comittee.

(c) Here, Out of 5 Mathematician Two Mathematician is remove. So, We have to select 2 Mathematician out of 3 Mathematician.

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Number of Selection $= 3C_2 \times 7C_3$

$= 105$

There are 105 ways to select committee.

6 A Group consist of 4 girls and 7 boys. In how many ways can a team of 5 memeber be selected if the team has,

(a) no Girl
(b) at least one boy and one Girl
(c) at least 3 Girls.

(a) There are 4 Girls and 7 Boys and we have to select 5 member but no Girl.

Number of Selection $= 7C_5$

$= 21$

There are 21 way to select 5 member.

(b) Here, we have to select atleast one boy and Girl.

Boy	Girl
1	4
2	3
3	2
4	1

ber

Number of Selection $= 7C_1 \times 4C_4 + 7C_2 \times 4C_3$

$7C_3 \times 4C_2 + 7C_4 \times 4C_1$

$= 441$

There are 441 ways to select 5 member.

(c) Here, we have to select atleast 3 Girl.

Boy	Girl
2	3
1	4

Number of Selection $= 7C_2 \times 4C_3 + 7C_1 \times 4C_4$

$= 91$

There are 91 way to select 5 member.

7 In how many ways can a teacher choose one or more students from six eligible students?

Here, There are six student and we have to select one or more student.

Number of Selection $= 6C_1 + 6C_2 + 6C_3 + 6C_4 + 6C_5 + 6C_6$

$= 63$

There are 63 way to select one or more student.

* Task : 4 : Probability

1 Define the following terms:

(a) Sample Space:

The set of all Possible outcomes of a random experiment is called Sample space.

It is denoted by S.

(b) Sample:

When we do a random experiment and we obtain one specific value out of the set of its possible values is called sample.

(c) Event:

Any subset of E of a Sample space is called event.

(d) Elementary event:

If an event E has only one sample point in sample space is called ~~sa~~ elementary event.

Q2 Let a card be selected at random from an 52 cards.
Let A = the card is spade.
B = the card is face.
What is $P(A)$, $P(B)$ and $P(A \cap B)$

Here, Given Sample space is 52 cards.

Event A = Card is Spade
B = Card is Face card.

There are 13 Spade card in 52 cards.

$$\therefore P(A) = \frac{13}{52} = \frac{1}{4}$$

There are 12 Face card in 52 cards.

$$\therefore P(B) = \frac{12}{52} = \frac{3}{13}$$

There are 3 comman card in event A and B.

$$\therefore P(A \cap B) = \frac{3}{52}$$

3 Three lighs bullbs are chosen at random from 15 bulls of which 5 are defective. Find the Probability that a) none is defective b) exactly one is defective c) at least one is defective.

Let Sample Space S = 15 ~~Bubls~~ Bullbs.

Event A : none is defective
B : Exactly one is defective
C : At least one is defective.

There are Total 15 bullbs and in which Five are defective.

$$P(A) = \frac{10C_3}{15C_3} = \frac{24}{91}$$

$$P(B) = \frac{5C_1 \times 10C_2}{15C_3} = \frac{45}{91}$$

$$P(C) = \frac{5C_1 \times 10C_2 + 5C_2 \times 10C_1 + 5C_3 \times 10C_0}{15C_3}$$

$$= \frac{67}{91}$$

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4 A lot comprises 10 good article, 4 with minor defects and 2 with major defects. Two article chosen. Find the Probability that a) Both are good b) Both have major defects c) at least one is Good d) At most one is good e) exactly one is good f) neither has major defect g) neither is good.

Let Sample Space S = 16 article

A: Both are Good
B: Both have major defects
C: Atleast one is Good
D: Atmost one is Good
E: Exactly one is Good
F: Neither has major defect
G: Neither is Good.

Total 10 articale are Good.

So, $P(A) = \frac{10C_2}{16C_2} = \frac{3}{8}$

Total 2 article has major defects.

So, $P(B) = \frac{2C_2}{16C_2} = \frac{1}{120}$

Atleast one is Good means more than one

$$P(C) = \frac{10C_1 \times 6C_1 + 10C_2 \times 6C_0}{16C_2}$$

$$= \frac{7}{8}$$

Atmost one is Good means less than one.

$$P(D) = \frac{10C_1 \times 6C_1 + 6C_2}{16C_2}$$

$$= \frac{5}{8}$$

Exactly one is Good means only one is good.

$$P(E) = \frac{10C_1 \times 6C_1}{16C_2}$$

$$= \frac{1}{2}$$

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Neither has major defect means both are Good.

$$P(F) = \frac{10C_2}{16C_2} = \frac{91}{120}$$

Neither is Good means article is defect or major defect.

$$P(G) = \frac{8C_2}{16C_2} = \frac{1}{8}$$

5 A Ball Drawn at random from a box containing 6 red Ball, 4 white Ball and 5 blue ball. Determine the Probability that is
a) Red b) White c) Blue
d) not red e) Red or white.

Let, Given Sample space is 15 Ball.

There are 6 Red, 4 White and 5 blue Ball.

A : Ball is Red
B : Ball is white
C : Ball is Blue

D : Ball is not red
E : Ball is Red or white.

There are 6 Red Balls.

$\therefore P(A) = \frac{6}{15} = \frac{2}{5}$

There are 4 white Balls.

$\therefore P(B) = \frac{4}{15}$

There are 5 Blue Balls

$\therefore P(C) = \frac{5}{15} = \frac{1}{3}$

There are not red Balls means there are blue and white Balls.

$\therefore P(D) = \frac{9}{15} = \frac{3}{5}$

There are 10 Red or white Balls.

$\therefore P(E) = \frac{10}{15} = \frac{2}{3}$

6 If a fair coin is tossed 4 times. define the sample space corresponding to this random experiment. Find the Probabilities a) More heads than tail b) Tails occur on the even numbered toses.

Let Givien Sample space element is $2^4 = 16$

S = { HHHH, HHHT, HHTH, HTHH,
THHH, HHTT, THHT, HTTH,
TTHH, HTTT, TTTH, TTTT,
HTHT, THTH, THTT, TTHT }

A : more head than tail

A = { HHHH, HHHT, HHTH,
HTHH, THHH }

$P(A) = \frac{5}{16}$

B : Tails occur on the even numbered toses.

B = { HTHT, TTHT, TTTT, HTTT }

$P(B) = \frac{4}{16} = \frac{1}{4}$

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7 What is the chance that a non-leap year should have Fifty three Sundays?

Let, Non-leap Year contain 365 days in Year.

So, There are 52 week in one Year and 1 day is extra.

There 1 day can be any day of the week.

So, Sample Space = 7 Day.

A : Fifty Three Sundays.

So, $P(A) = \frac{1}{7}$

8 A man and his wife apper for an interview for two posts. The Probability of the husband's selection is 1/7 and that of the wife's selection is 1/5. What is the Probability that only of them will be selected?

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Let A : Husband is select
B : WiFe is select

So, $P(A) = \frac{1}{7}$, $P(B) = \frac{1}{5}$

Probability of wife does not e select $P(B') = 1 - \frac{1}{5}$

$= \frac{4}{5}$

Probability of husband is not select $P(A') = 1 - \frac{1}{7}$

$= \frac{6}{7}$

C : Only of them will be select.

$\therefore P(C) = P(A) \cdot P(B') + P(B) \cdot P(A')$

$= \frac{1}{7} \times \frac{4}{5} + \frac{1}{5} \times \frac{6}{7}$

$= \frac{2}{7}$

* Task : 5 : Conditional Probability

1 Write definition of conditional Probability. A lot of 100 keyboard contain 20 that are defective. Two keyboards are selected at random without replacement from the lot,

a What is the Probability that the First one selected is defective?

b What is the Probability that the second one selected is defective given that the first one was defective?

c Both are defective?

=> Conditional Probability :

Conditional Probability is known as the possibility of an event happening based on the existence of a previous event.

=> Let A : First one is Defective
B : Second one is Defective.

a $P(A) = \frac{20}{100} = \frac{1}{5} = 0.2$

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b $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$= \frac{\frac{20}{100} \times \frac{19}{99}}{0.2}$

$= 0.19$

c $P(B \cap A) = \frac{20}{100} \times \frac{19}{99}$

$= 0.03838$

2 A Box of 100 gasket contain 10 gasket with type A defects, 5 gasket with type B defects, 2 gasket with both type of defects. Find the Probability that,

a a gasket to be drawn has a type B defect under the condition that it has a type A defects.

b a gasket to be drawn has no type B defect under the condition that it has no type A defects.

Let A : A Gasket have A type Defects.

B : A Gasket have B type Defects.

So, $P(A) = \frac{10}{100} = 0.1$

$P(B) = \frac{5}{100} = 0.05$

$P(A \cap B) = \frac{2}{100} = 0.02$

a $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{0.02}{0.1}$

$= 0.2$

b $P\left(\frac{B'}{A'}\right) = \frac{1 - P(A \cup B)}{P(A)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.1 + 0.05 - 0.02$

$= 0.13$

$P(A') = 1 - P(A)$
$= 1 - 0.1$
$= 0.9$

$$P\left(\frac{B'}{A'}\right) = \frac{1 - P(A \cup B)}{P(A')}$$

$$= \frac{1 - 0.13}{0.9}$$

$= 0.966$

3 Let A and B be events with $P(A) = 1/2$, $P(B) = 1/8$ and $P(A \cap B) = 1/4$ Find a) $P(A/B)$ b) $P(B/A)$ c) $P(A \cup B)$ D) $P(A'/B')$ e) $P(B'/A')$

$\Rightarrow$ Let Given, $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{8}$

$P(A \cap B) = \frac{1}{4}$

$P(A') = 1 - \frac{1}{2} = 0.5$

$P(B') = 1 - \frac{1}{8} = 0.875$

a $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{1/8}$

$= 2$

b $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$= \frac{1/4}{1/2}$

$= \frac{1}{2}$

c $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{8} - \frac{1}{4}$

$= \frac{3}{8}$

D $P\left(\frac{A'}{B'}\right) = \frac{1 - P(A \cup B)}{P(B')}$

$= \frac{1 - 3/8}{0.875} = \frac{5}{7}$

c $P\left(\frac{B'}{A'}\right) = \frac{1 - P(A \cup B)}{P(A')}$

$= \frac{1 - 3/8}{0.5}$

$= \frac{5}{4}$

4 A teacher gave her students of two class testly namely maths and science. 25 % of the student passed both test and 40 % of the student passed maths test. What percent of those who passed the maths test also passed the science test?

Let A : Student Passed maths Test

B : Student Passed Science Test.

$P(A) = \frac{40}{100} = 0.4$

$P(A \cap B) = \frac{25}{100} = 0.25$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.25}{0.4}$$

$$= 0.625$$

So, there are 62.5 % student pass maths test and also pass science test.

5, A Pair of fair dice is thrown. Find the Probability that the sum is 10 greater if

a a 5 appers on first die

b a 5 appers on at least one of the die.

Here, There are Two dice is thrown.

A : Sum is greater or 10.
B : 5 appers on the die.

$A = \{(5,5), (5,6), (6,5), (6,4), (4,6), (6,6)\}$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

a B_1 : 5 apperar on First dice

$B_1 = \{(5,5), (5,6)\}$

$P(B_1) = \frac{2}{36} = \frac{1}{18}$

$P\left(\frac{B_1}{A}\right) = \frac{P(A \cap B_1)}{P(A)}$

$= \frac{2/36}{1/6}$

$= \frac{1}{3}$

b B_2 : 5 appear on at least one of the dice

$B_2 = \{(1,5), (2,5), (3,5), (4,5), (5,5), (5,6), (6,5), (5,4), (5,1), (5,2), (5,3)\}$

$P(B_2) = 11/36$

$\cancel{P\left(\frac{B_2}{A}\right)} = P\left(A/B_2\right) = \frac{P(A \cap B_2)}{P(B_2)}$

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$$= \frac{3/36}{11/36}$$

$$= \frac{3}{11}$$

6 In a certain college, 25% of the student failed Math, 15% of the student failed chemistry and 10% of the student failed in both. A student is selected at random.

(a) If he failed chemistry, what is the Probability that he failed in Math?

(b) If he failed maths, what is the Probability that he failed in chemistry?

(c) he failed maths or chemistry?

Let, A : Failed in Maths
B : Failed in Chemistry.

$$P(A) = \frac{25}{100} = 0.25$$

$$P(B) = \frac{15}{100} = 0.15$$

$$P(A \cap B) = \frac{10}{100} = 0.1$$

(a) $P\left(A/B\right) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.1}{0.15}$

$= \dfrac{2}{3}$

(b) $P\left(B/A\right) = \dfrac{P(A \cap B)}{P(A)}$

$= \dfrac{0.1}{0.25}$

$= \dfrac{2}{5}$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.15 - 0.1$

$= 0.3$

* Task : 6 : Baye's Theorem:

1 Write Baye's Theorem and Three urns contain 6 green, 4 black; ~~and~~ 4 green, 6 black and 5 green 5 black balls respectively. Randomely selected an urns and a ball is drawn. If the ball drawn is Green then Find the Probability that it is drawn from the First urn.

=) Baye's Theorem:

Let S be a Sample spaces and $E_1, E_2, \ldots, E_n$ be a mutualy exclusive and exhaustive event than Probability of E_i ocuures when A is already ocuures.

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i \cap A)}{P(A)}$$

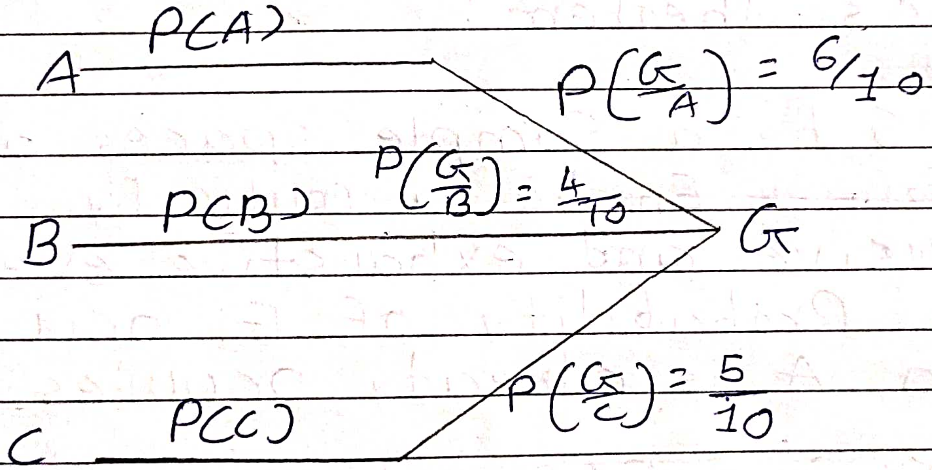
$$= \frac{P(E_i) \cdot P(A/E_i)}{\cancel{P(A)} \; \sum_{i=0}^{n} P(E_i) \cdot P(A/E_i)}$$

$\Rightarrow$ Here, There are Three urns.

So, A : Ball from First urns
B : Ball from Secound urns
C : Ball from third urns
G : Green Ball.

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{3}$

$P(C) = \frac{1}{c}$



$$P\left(\frac{A}{G}\right) = \frac{P(G \cap A)}{P(A) \cdot P\left(\frac{G}{A}\right) + P(B) \cdot P\left(\frac{G}{B}\right) + P(C) \cdot P\left(\frac{G}{C}\right)}$$

$$= \frac{P(A) \cdot P(G/A)}{P(A) \cdot P\left(\frac{G}{A}\right) + P(B) \cdot P\left(\frac{G}{B}\right) + P(C) \cdot P\left(\frac{G}{C}\right)}$$

$$P\left(\frac{A}{G}\right) = \frac{1/3 \times 6/10}{\frac{1}{3}\times\frac{6}{10} + \frac{1}{3}\times\frac{4}{10} + \frac{1}{3}\times\frac{5}{10}}$$

$$= \frac{2}{5}$$

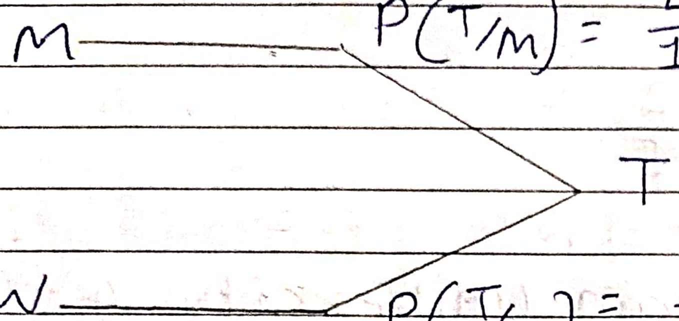
7 At a certain university 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected random from among all those over six feet tall. What is the Probability that the student is a women?

Here, There are two Category.

So, M : Student from Men
W : Student from Women
T : Student tall over six feet.

Total Persentage $= 4 + 1$
$= 5$

$P(M) = \frac{2}{5}$ ~~PEN~~ $P(W) = \frac{3}{5}$



$P(T/M) = \frac{4}{100} = 0.04$

$P(T/W) = \frac{1}{100} = 0.01$

$$P\left(W/T\right) = \frac{P(T \cap W)}{P(T)}$$

$$= \frac{P(W) \cdot P\left(T/W\right)}{P(M) \cdot P\left(\frac{T}{M}\right) + P(W) \cdot P\left(\frac{T}{W}\right)}$$

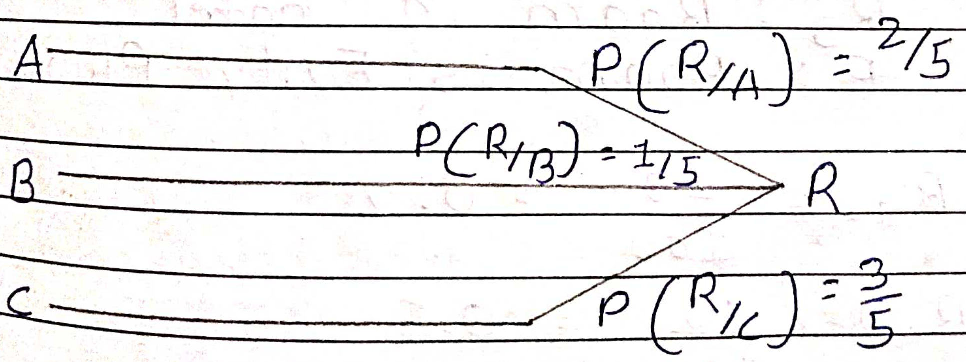
$$= \frac{3/5 \times 0.01}{3/5 \times 0.01 + 2/5 \times 0.04}$$

$$= \frac{3}{11}$$

3 Three boxes A, B and C contain red and black balls. Box A contain 2 red and 3 black balls. Box B contain 1 red and 4 black balls. Box C contain 3 red and 1 black ball. If a student randomaly choose a box and from this box contain red ball and the ball comes from box A.

Let A : Ball from Box A
B : Ball from Box B
C : Ball from Box C.
R : Ball is Red.

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{3}$



$$P\left(A/R\right) = \frac{P(A) \cdot P\left(R/A\right)}{P(A) \cdot P\left(R/A\right) + P(B) \cdot P\left(R/B\right) + P(C) \cdot P\left(R/C\right)}$$

$$P\left(\frac{A}{R}\right) = \frac{1/3 \cdot 2/5}{\frac{1}{3}\cdot\frac{2}{5} + \frac{1}{3}\cdot\frac{1}{5} + \frac{1}{3}\cdot\frac{3}{5}}$$

$$= \frac{8}{27}$$

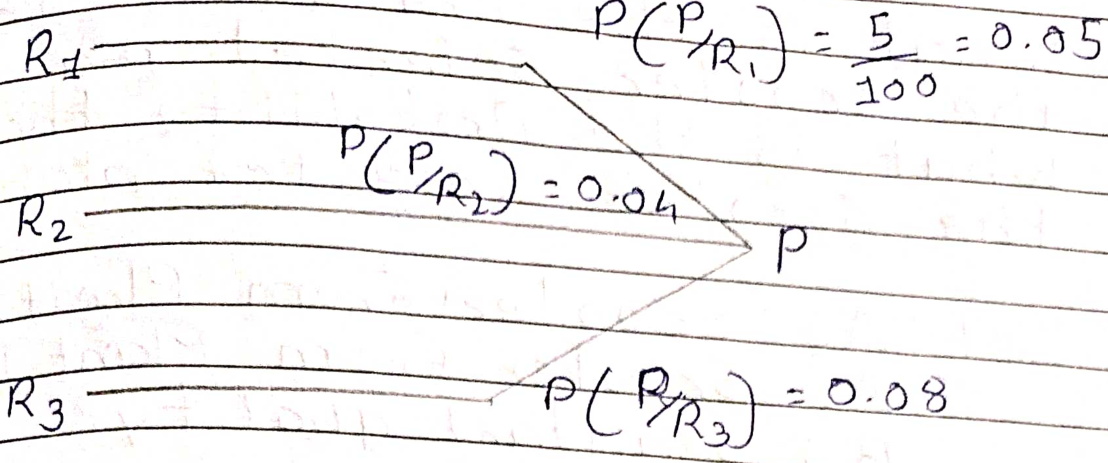
4 A business man goes to hotels X, Y and Z by 20%, 50% and 30% of the time. If 5%, 4% and 8% of the rooms in the hotel X, Y and Z have fully plumbing. Find the probability that the business man room having fully plumbing is to hotel Z.

Let R_1 : Room in hotel X
R_2 : Room in hotel Y
R_3 : Room in hotel Z
P : Room is Fully Plumbing.

$P(R_1) = \frac{20}{100} = 0.2$

$P(R_2) = \frac{50}{100} = 0.5$

$P(R_3) = \frac{30}{100} = 0.3$



$$P\left(\frac{R_3}{P}\right) = \frac{P(R_3) \cdot P(P/R_3)}{P(R_1) \cdot P(P/R_1) + P(R_2) \cdot P(P/R_2) + P(R_3) \cdot P(P/R_3)}$$

$$= \frac{0.3 \times 0.08}{0.3 \times 0.08 + 0.5 \times 0.04 + 0.2 \times 0.05}$$

$$= 0.44$$

5 A company has two plants to manufacture scooters. Plant I manufacture 80% of the scooter and II manufacture 20%. At plant I, 25 out of 100 scooter are rated standard quality and plant II only 65 out of 100 scooter are rated standard quality.

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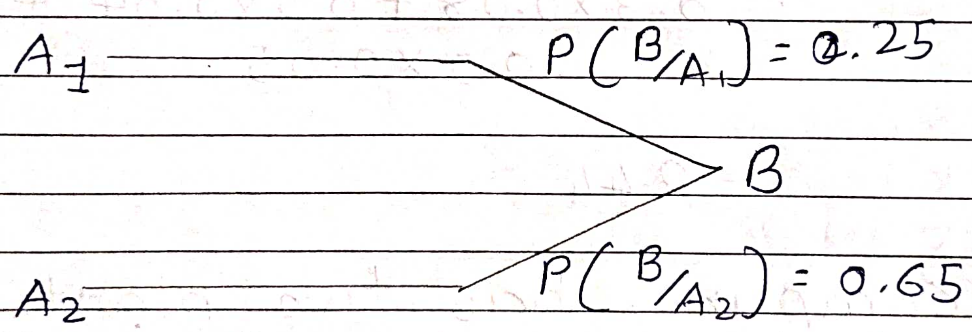
a What is the Probability that the scooter selected plant I.

b What is the Probability that the Scooter selected plant II

Let A_1 : Scooter from Plant I
A_2 : Scooter from Plant II
B : Standard quality scooter.

$$P(A_1) = \frac{80}{100} = 0.8$$

$$P(A_2) = \frac{20}{100} = 0.2$$



a $$P(A_1/B) = \frac{P(A_1) \cdot P(B/A_1)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)}$$

$$= \frac{0.8 \times 0.25}{0.8 \times 0.25 + 0.2 \times 0.65}$$

$P\left(\frac{A_1}{B}\right) = 0.60$

b $P\left(\frac{A_2}{B}\right) = \dfrac{P(A_2) \cdot P(B/A_2)}{P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)}$

$= \dfrac{0.65 \times 0.2}{0.65 \times 0.2 + 0.8 \times 0.25}$

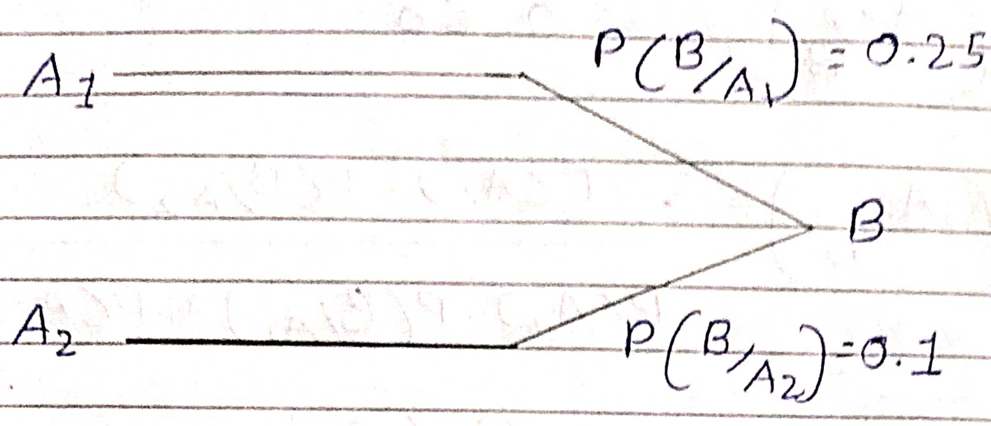
$= 0.39$

6 In a certain college 25% of boys and 10% of girls are study maths. The girls consitute 60% of the students. If a student is selected at random and is found to be study maths find the Probability that the student is a a) Girl b) Boy.

Let A_1 : Student is Girl
A_2 : Student is Boy
B : Student is study maths.

$P(A_1) = \dfrac{40}{100} = 0.4$

$P(A_2) = \dfrac{60}{100} = 0.6$



a

$$P(A_2/B) = \frac{P(A_2)\cdot P(B_2/B_2)}{P(A_2)\cdot P(B/B_2) + P(A_1)\cdot P(B/B_1)}$$

$$= \frac{0.6 \times 0.1}{0.6\times 0.1 + 0.4 \times 0.25}$$

$$= \frac{3}{8}$$

b

$$P(A_1/B) = \frac{P(A_1)\cdot P(B/A_1)}{P(A_1)\cdot P(B/A_1) + P(A_2)\cdot P(B/A_2)}$$

$$= \frac{0.4 \times 0.25}{0.6\times 0.1 + 0.4\times 0.25}$$

$$= \frac{5}{8}$$

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7 A Factory production line is manufacturing bolts using three machines. A, B and C. Of the machine A is responsible of 25% machine B is 35% and machine C is for the rest. It is known from perivious experience with the machine that 5% of the output from machine A is defective. 4% from machine B and 2% from machine C. A bolt is chosen and bolt is defective. What is Probability that it came from a) machine A b) machine B c) machine C.

B/B_1

Let, A_1 : Bolt from machine A
A_2 : Bolt from machine B
A_3 : Bolt from machine C
B : Bolt is Defective.

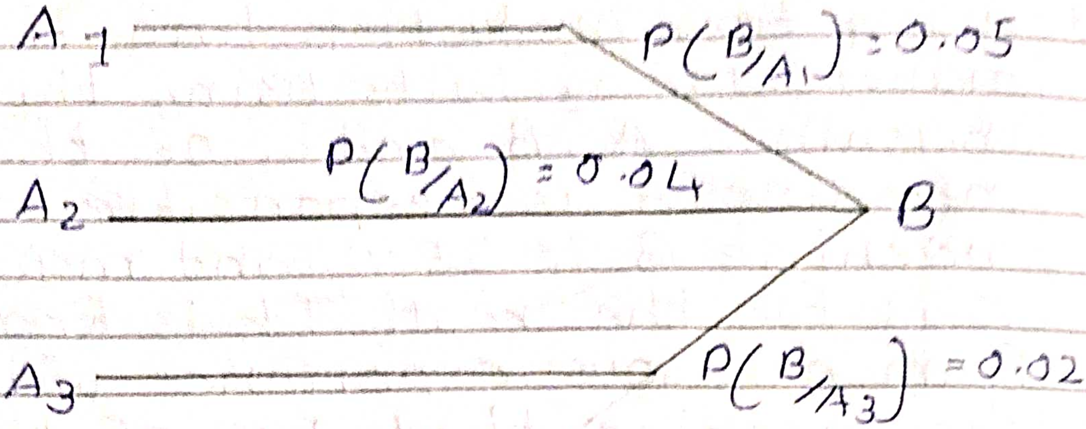
B/A_2

$$P(A_1) = \frac{25}{100} = 0.25$$

$$P(A_2) = \frac{35}{100} = 0.35$$

$$P(A_3) = \frac{100-35+25}{100} = \cancel{0.64}\ \cancel{0.6}$$

$$= 0.4$$



a Machine A

$$P\left(\frac{A_1}{B}\right) = \frac{P(A_1)\cdot P(B/A_1)}{P(A_1)\cdot P(B/A_1) + P(A_2)\cdot P(B/A_2) + P(A_3)\cdot P(B/A_3)}$$

$$= \frac{0.25 \times 0.05}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02}$$

$$= 0.362$$

b Machine B

$$P\left(\frac{A_2}{B}\right) = \frac{P(A_2)\cdot P(B/A_2)}{P(A_1)\cdot P(B/A_1) + P(A_2)\cdot P(B/A_2) + P(A_3)\cdot P(B/A_3)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02}$$

$$= 0.406$$

c Machine C

$$P\left(\frac{A_3}{B}\right) = \frac{P(A_3) \cdot P(B/A_3)}{P(A_3) \cdot P(B/A_3) + P(A_1) \cdot P(B/A_1) + P(A_2) \cdot P(B/A_2)}$$

$$= \frac{0.4 \times 0.02}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02}$$

$$= 0.232$$

* Task : 7 : Independent Event

1 Probability that a man will be alive 25 years hence is 0.3 and the Probability that his wife alive 25 years hence is 0.4. Find the Probability that 25 Years hence a) Both will alive b) only the man will alive c) only wife will alive d) none will alive e) aleast one them will alive.

Let A : Men will be alive
B : Wife will be alive.

$\therefore P(a) = 0.3, \quad P(B) = 0.4$

a

$$P(A \cap B) = P(A) \cdot P(B)$$
$$= 0.3 \times 0.4$$
$$= 0.12$$

b

$$P(A \cap B') = P(A) \cdot P(B')$$
$$= 0.3 \cdot (1 - P(B))$$
$$= 0.3 \cdot (1 - 0.4)$$
$$= 0.18$$

C

$P(A' \cap B) = P(A) \cdot P(B)$
$= (1 - P(A)) \cdot P(B)$
$= (1 - 0.3) \cdot 0.4$
$= 0.28$

D

$P(A' \cap B') = P(A') \cdot P(B')$
$= 0.7 \cdot 0.6$
$= 0.42$

e

$P(A \cap B) = 1 - P(A' \cap B')$
$= 1 - 0.42$
$= 0.58$

2 A Problem in statistics is given to three student A, B and C. whose chances of solving it are 1/3, 1/4 and 1/5 respectively. Find the Probability that the solve if they all try independently.

Let, A : Student A occurs Problem
B : Student B occurs Problem
C : Student C occurs Problem

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{5}$

$$P(A') = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(B') = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(C') = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P(A \cup B \cup C) = 1 - [P(A') \cdot P(B') \cdot P(C')]$$

$$= 1 - \left[\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}\right]$$

$$= \frac{3}{5}$$

3 Find the Probability of throwing 6 at least once in six throws with a single die.

Here, There are six throws with a single die.

A : Get 6 in Single throw

$$\therefore P(A) = \frac{1}{6}$$

$P(A') = 1 - \frac{1}{6} = \frac{5}{6}.$

Probability For Get 6 atleast six throw $= (P(A'))^6$

$= \left(\frac{5}{6}\right)^6$

$= 0.334897$

4 Two Fair dice, One colored white and One colored red are thrown, Find the Probability that

a The Score on red is 2 and white die is 5.

b The Score on the white die is 1 and Red is even.

a W : White die is 5
R : Red die is 2

$P(W) = \frac{1}{6}$, $P(R) = \frac{1}{6}$

Both event are mutualy independent.

So, $P(W \cap R) = P(W) \cdot P(R)$

$= 1/36$

6 W : White die is 1
R : Red die is even

$P(W) = \frac{1}{6}$, $P(R) = \frac{3}{6} = \frac{1}{2}$

Both event are mutualy independant,

$P(W \cap R) = P(W) \cdot P(R)$

$= \frac{1}{6} \cdot \frac{1}{2}$

$= \frac{1}{12}$

5 If A and B are the two possible outcomes of a random experiment such that $P(A) = 0.4$, $P(A \cup B) = 0.7$ and $P(B) = P$ then,

a For what choice of P, A and B are mutually exclusive?

b For what choice of P, A and B are independant?

Given, $P(A) = 0.4$
$P(B) = P$
$P(A \cup B) = 0.7$

a For Mutualy Exclusive event,

$P(A \cup B) = P(A) + P(B)$
$\therefore 0.7 = 0.4 + P$
$\therefore P = 0.3$

$\therefore P(B) = 0.3$

b For Independant event,

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$\therefore P(A) + P(B) - P(A \cup B) = P(A) \cdot P(B)$

$\therefore 0.4 + P - 0.7 = 0.4P$

$\therefore P - 0.4P = 0.7 - 0.4$

$\therefore 0.6P = 0.3$

$\therefore P = 0.5$

$\therefore P(B) = 0.5$

6 The Probability that A hits a target is 1/4 and the Probability that B hits it is 2/5. What is the Probability that the target will be hit if A and B each shoot at the target?

Let A : A hits a target
B : B hits a target

$$P(A) = \frac{1}{4}, \quad P(B) = \frac{2}{5}$$

$$P(A') = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(B') = 1 - \frac{2}{5} = \cancel{4}\ \frac{3}{5}$$

$$P(A \cup B) = 1 - (P(A') \cdot P(B'))$$

$$= 1 - \left(\frac{3}{4} \cdot \frac{3}{5}\right)$$

$$= 1 - \frac{9}{20}$$

$$= \frac{11}{20}$$

7 The Probability that a man will live 10 more years is 1/4 and the Probability that his wife will live 10 more years is 1/3. Find the Probability that,

a Both will be alive in 10 Year

b atleast one will be alive in 10 Year

c neither will be alive in 10 Year

d only the wife will be alive.

Let, M : Men will alive 10 Year
W : Wife will alive 10 Year.

$P(M) = \frac{1}{4}$, $P(W) = \frac{1}{3}$

$P(M') = 1 - \frac{1}{4} = \frac{3}{4}$

$P(W') = 1 - \frac{1}{3} = \frac{2}{3}$

a

~~$P(A \cap B) = P$~~

$P(M \cap W) = P(M) \cdot P(W)$

$= \frac{1}{4} \times \frac{1}{3}$

$= 1/12$

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b $P(W \cup M) = 1 - P(M' \cap W')$

$= 1 - (P(M') \cdot P(W'))$

$= 1 - \frac{3}{4} \times \frac{2}{3}$

$= \frac{1}{2}$

c ~~P(M~~

$P(M' \cap W') = P(M') \cdot P(W')$

$= \frac{3}{4} \times \frac{2}{3}$

$= \frac{1}{2}$

d

~~$P(W' \cap M')$~~

$P(W \cap M') = P(M') \cdot P(W)$

$= \frac{3}{4} \times \frac{1}{3}$

$= \frac{1}{4}$