Unit : 1 : Introduction

* Set : Set is a define as a collection of Object.

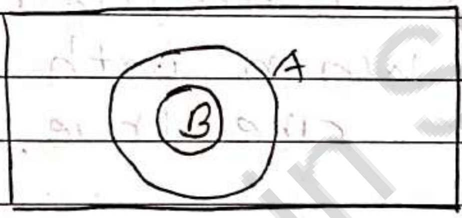
Ex. $A = \{1, 2, 3, 4, 5\}$

* Subset :

Symbol : $\subseteq$

Ex. $A = \{1, 2, 3\}, \quad B = \{2\}$

$A \subseteq B$



* Complement : The complement of a set is called those element which are not in a set but this is element of Universal set U.

Ex. $A = \{1, 2, 3\}, \quad U = \{1, 2, 3, 4, 5\}$

$A' = \{4, 5\}$

* Set Difference : A - B of two sets A and B is the set ~~that~~ of everything in A but not in B.

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Ex. $A = \{1, 2, 3\}, \quad B = \{2\}$

$A - B = \{1, 3\}$

* Symmetric Difference :

Ex. $A = \{1, 2, 3\} \quad B = \{2\}$

$$A \ominus B = (A - B) \cup (B - A)$$
$$= \{1, 3\} \cup \{\emptyset\}$$
$$= \{1, 3\}$$

* Conjuction : Conjuction is true only when both of the Propositions are true.

Truth Table :

p	q	p ∧ q
F	F	F
F	T	F
T	F	F
T	T	T

* Disjunction : Disjunction is true if one or both of the Propositions are true.

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p	q	p ∨ q
F	F	F
F	T	T
T	F	T
T	T	T

* Negation:

p	q	$\neg p$	$\neg q$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	F	F

* Relation:

1 Reflexive:

If aRA => aRa

2 Symmetric: a, b ∈ A

if aRb then bRa

3 Transitive: a, b, c ∈ A

if aRb, bRc then aRc

If Set A Contains all the

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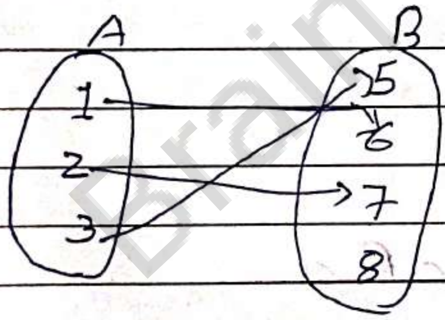
Relation so, Set A is called Equivalence Relation.

* Function:

1 One to One Function:

A Function for which every element of the range of the Function corresponds to exactly one element of the domain is known as a One-to-One Function.

Ex.



2 Onto Function:

Set A and Set B, which consist of elememts, If for every element of B there is at least one or more than One element matching with A.

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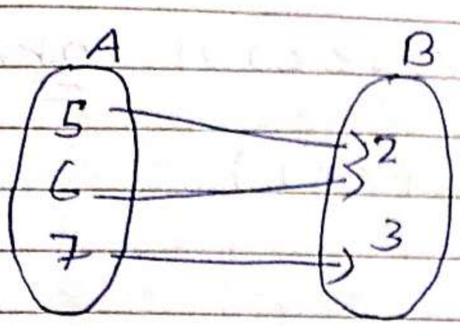
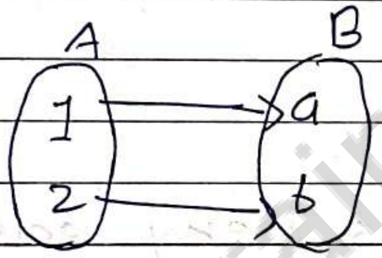
Ex.



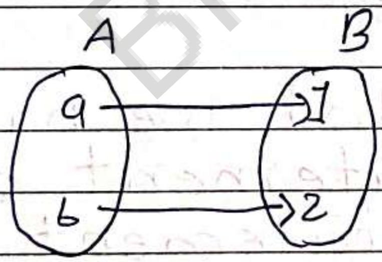
* Inverse of Function:

If Function, domain and Range is Set and It is Invertible if there exists a Function with domain and Range.

Ex.



$F^{-1} \rightarrow$



* Composition Function:

Let, Function $F(x) = ax + b$
Function $g(x) = cx + d$

So, Composition Function,

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$f o g = f(g(x))$ or

$g o f = g(f(x))$

* **Proof Tech. and Types :**

There are ~~three~~ four Proof Techniques.

1) Direct Proof
2) Proof by Contradiction
3) Proof by Induction
4) Proof by Contrapositive

1 **Direct Proof :**

Direct Proof is very easy and simple method.

In this method, we have to take One statement as P and Second statement as q.

In this method, we have to assumes P is always true and using P we have to Prove q is also true.

$\therefore P \rightarrow q$

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Ex. If a and b are odd integer number than sum of a and b is even.

-> Here, We have to take P and q statement.

So, P = a and b are odd integer
q = Sum of a and b is even.

Using Direct Proof, Assumes that P is always true.

Assume, $a = 2k+1$, $b = 2i+1$

For q statement, sum of a and b is even.

$\therefore\ a+b = (2k+1) + 2i+1$

$\therefore\ a+b = 2(k+i+1)$

Assume $k+i+1 = c$

$\therefore\ a+b = 2c$

Here, 2c is always become even number.

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Hence, Proof, $P \rightarrow q$, Sum of two odd number is always even.

2 Proof by Contradiction:

In this method, we have to take one statement P and second statement q.

In this method, We have to take P is always true and take q is always False.

$$P \rightarrow \sim q$$

Ex. If a and b are odd integer number than sum of a and b is even.

-> Here, we have to take P and q statement.

So, P = a and b are odd integer number
q = Sum of a and b is even.

According to Contradiction method we have to take

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$\sim q \rightarrow$ Sum of a and b is odd.

We have to Prove, $p \rightarrow \sim q$

Assume $a = 2k + 1$, $b = 2i + 1$

$\therefore a + b = 2k + 1 + 2i + 1$

$= 2(k + i + 1)$

Assume $k + i + 1 = c$

$\therefore a + b = 2c$

Here, 2c is always become even number not odd number.

So, Hence Prove, sum of two odd number is always even.

3 Proof By Induction:

In this method, we have to follow three steps.

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(i) Basis Step: Take $n = 0$ or 1 and Prove it is True For $P(n)$

(ii) Induction Hypothesis: Prove $P(n)$ is ~~prove~~ true For any k integer.

(iii) Induction Step: Prove $P(n)$ is true for any $n = k+1$

Ex $P(n) = 1+2+3+\dots+n = \frac{n(n+1)}{2}$

By Induction Method,

- Step-1: Basis step,

For $n = 0$

$L.H.S = 1+2+3+\dots+0$

$= 0$

$R.H.S = \frac{n(n+1)}{2} = 0$

Here, $L.H.S = R.H.S$

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- Step-2 : Induction Hypothesis :

take $n = k$,

$\therefore$ L.H.S. $= 1 + 2 + 3 + \dots + k$

$$= \frac{k(k+1)}{2} \quad - (1)$$

$\therefore$ R.H.S $= \frac{n(n+1)}{2} = \frac{k(k+1)}{2} \quad - (2)$

$\therefore$ L.H.S. = R.H.S.

- Step-3 : Induction Step,

take $n = k+1$,

$\therefore$ L.H.S $= 1 + 2 + 3 + \dots + k + k+1$

$$= \frac{k(k+1)}{2} + k+1$$

$$= \frac{(k^2 + k)}{2} + k + 1$$

$$= \frac{k^2 + k + 2k + 2}{2}$$

$$= \frac{(k+1)^2 + (k+1)}{2}$$

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$\therefore$ R.H.S. $= \frac{n^2+n}{2}$

$= \frac{(k+1)^2+(k+1)}{2}$

$\therefore$ L.H.S. = R.H.S.

Hence, Prove $p(n) = 1+2+3+\dots+n$

$= \frac{n(n+1)}{2}$

4 Proof by Contrapositive.

In this method, we have to take one statement as p and second statement as q.

In this method, we have to Prove p is False and q is also False.

$\therefore \sim p \rightarrow \sim q$

Ex. If a and b are odd integer number than sum of a and b is even.

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→ Here, We have to take p and q Statement.

∴ p → a and b are odd
q → Sum of a and b is even.

According to Contrapositive,

∴ ~p → a and b are even
∴ ~q → Sum of a and b is odd.

We have to Prove ~p → ~q

Assume $a = 2k$ ~~+1~~, $b = 2i$

∴ $a + b = 2k + 2i$

$= (k + i)2$

Assume $k + i = c$

∴ $a + b = 2c$

Here, 2c is become even For every integer number.

Hence Prove, a and b are odd integer than sum of a and b is even.

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Ex. Let $X = \{1,2,3,4,5,6,7\}$ and $R = \{(x,y) / x-y$ is divisible by $3\}$ then show that R is an equivalence relation.

$\Rightarrow$ Here, Given Set $X = \{1,2,3,4,5,6,7\}$ and $R = \{(x,y) / x-y$ is divied by $3\}$

$R = \{(1,1), (1,4), (2,2), (3,3),$
$(4,4), (5,5), (6,6), (7,7),$
$(5,2), (2,5), (6,3), (3,6),$
$(7,4), (4,7), (5,6), (6,5)\}$

For Equivalence Relation, we have to check three relation,

(a) Reflexive Relation:

Let R be a Reflexive Relation, if $aRa, \forall a \in R$

$\therefore (a,a) \in R, \forall a \in R$

So, that R is Reflexive Relation.

(b) Symmetric Relation:

Let R be a Symmetric Relation,

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if $aRb \Rightarrow bRa$, $a, b \in R$

$\therefore (a, b) \in R \rightarrow (b, a) \in R$, $\forall a, b \in R$

So, that R is a Symmetric Relation.

(c) Transitive Relation:

Let R be a transitive Relation if aRb, $bRc \Rightarrow aRc$, $a, b, c \in R$

$\therefore (a, b) \in R$, $(b, c) \in R$
$\therefore (a, c) \in R$

So that, R is transitive relation.

Here, R is follow this three relation.

So, R is a Equivalence Relation.

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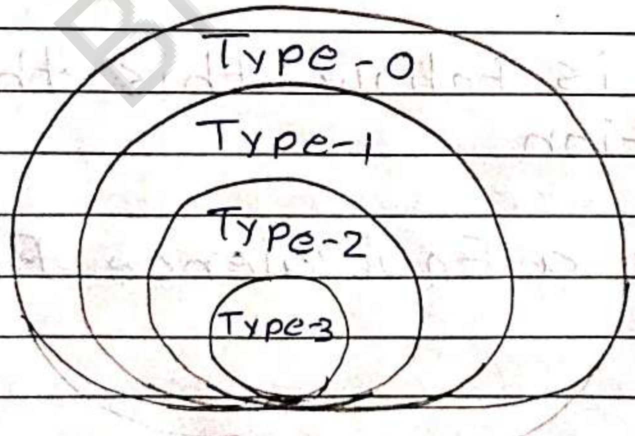
* Chomsky Hierarchy:

Chomsky Hierarchy is Used to represent the different types of Grammer.

Chomsky Hierarchy is Contains different types of Grammer.

There are Four types of Grammer

(i) Type - 0 Grammer
(ii) Type - 1 Grammer
(iii) Type - 2 Grammer
(iv) Type - 3 Grammer



(i) Type - 0 Grammer:

Type - 0 Grammer is also know as Unristricted Grammer.

This Grammer is recognized by Turing machine.

Using this Grammer, we can generate complex languages.

(ii) Type-1 Grammer:

Type-1 Grammer is also know as Context-Sensitive Grammer.

This Grammer is recognized by Linear Bound Automata.

Type-1 Grammer is only contains transformation of symbols.

(iii) Type-2 Grammer:

Type-2 Grammer is also know as Context-Free Grammer which is generate Context-Free languages.

This Language is recognized by Push Down Automata.

This Grammer is less Powerfull compare to Type 0 and Type 1 Grammer.

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(iv) Type -3 Grammer :

Type- 3 Grammer is know as Regular Grammer which is generate Regular Languages.

Type- 3 Grammer is the most restricted and less Powerfull Grammer.

- Type- 3 Grammer is recognized by Finite Automata.

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