

Unit : 5 : Dynamic Programming

* Explain Dynamic Programming with its characteristics and application.

=> Dynamic Programming is used to find optimal solution.

Dynamic Programming gives better solution to the Greedy Approach.

Dynamic Programming solution is always represented in tabular form.

- Characteristics :

- 1 The Problem can be divided into stages.
- 2 In Dynamic Programming, subproblem in a bottom-up fashion.
- 3 Solution are found using the tabular form.

4 ~~Dev~~ Divided Problem into part for find ~~small~~ optimal solution.

5 Principles of optimality allows to solve the problem in recursively.

- Application :

1 Used to Find 1/0 knapsack Problem.

2 Used to find Optimal Merge portions.

3 Used to find Shortest Path Problems.

4 Provides Mathematical Optimization.

5 Find Matrix chain multiplication sequence.

6 Used to find Longest common Subsequence.

* Explain Matrix Chain Multiplication Using Dynamic Programming.

=> Matrix Chain Multiplication is used to find matrix multiplication order.

Using this Problem, we can reduce the number of multiplication.

Method :

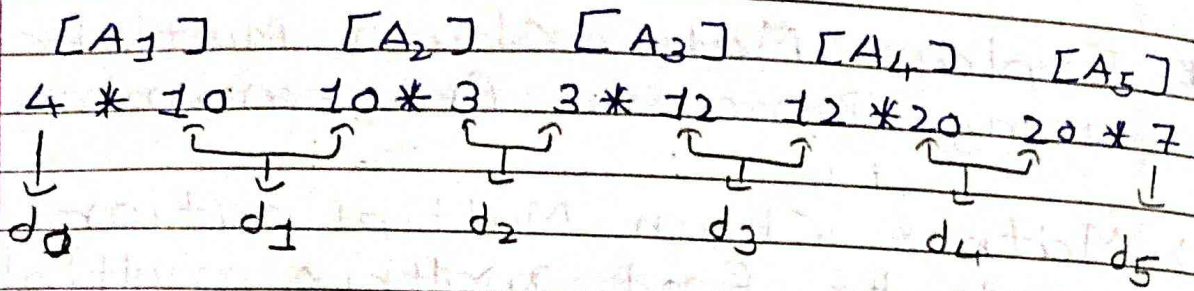
$$M[i, j] = \begin{cases} \text{Don't care } (x), & i > j \\ \min_{i \leq k < j} (M[i, k] + M[k+1, j] + d_{i-1} d_k d_j), & i = j \end{cases}$$

k = Value of i , which is increment for condition

Using this formula, we can find the solution.

Ex. $A_1 = 4 \times 10, A_2 = 10 \times 3, A_3 = 3 \times 12$
 $A_4 = 12 \times 20, A_5 = 20 \times 7$

For Finding the solution, we have to find value of d .



Hence, $d_0 = 4$, $d_1 = 10$, $d_2 = 3$,
 $d_3 = 12$, $d_4 = 20$, $d_5 = 7$

For finding the ~~table~~ solution we have to create table.

Here, There are 5 Matrix.
 So, We have to create 5x5 Matrix.

i/j	1	2	3	4	5
1	0	120	264	1080	1344
2	X	0	360	1320	1350
3	X	X	0	720	1140
4	X	X	X	0	1680
5	X	X	X	X	0

Using the Formula, we have to find the table value.

i	j	k	Condition	Value
1	1	1	$i = j, 1 = 1$	0
2	1	2	$i > j, 2 > 1$	X
3	1	3	$i > j, 3 > 1$	X
4	1	4	$i > j, 4 > 1$	X
5	1	5	$i > j, 5 > 1$	X
2	2	2	$i = j, 2 = 2$	0
3	2	3	$i > j, 3 > 2$	X
4	2	4	$i > j, 4 > 2$	X
5	2	5	$i > j, 5 > 2$	X
3	3	3	$i = j, 3 = 3$	0
4	3	4	$i > j, 4 > 3$	X
5	3	5	$i > j, 5 > 3$	X
4	4	4	$i = j, 4 = 4$	0
5	4	5	$i > j, 5 > 4$	X
5	5	5	$i = j, 5 = 5$	0

તીવ્ર ઘસાચી પથ્થરની દિવાલને પણ તોડી શકાય છે.

i	j	k	Condition	Value
1	2	1	$1 \leq 1 < 2$, So, Min $(M(1,1) + M(2,2) + d_0 d_1 d_2)$ $= \text{Min}(0 + 0 + 120) = 120$	120
		2	$1 \leq 2 < 2 \rightarrow \text{False}$	
2	3	2	$2 \leq 2 < 3$, So, Min $(M(2,2) + M(3,3) + d_1 d_2 d_3)$ $= 360$	360
		3	$2 \leq 3 < 3 \rightarrow \text{False}$	
3	4	3	$3 \leq 3 < 4$, So, Min $(\text{Min}(3,3) + M(4,4) + d_2 d_3 d_4)$ $= 720$	720
		4	$3 \leq 4 < 4 \rightarrow \text{False}$	
4	5	4	$4 \leq 4 < 5$, So, Min $(M(4,4) + M(5,5) + d_3 d_4 d_5)$ $= 1680$	1680
		5	$4 \leq 5 < 5 \rightarrow \text{False}$	
1	3	1	$1 \leq 1 < 3$, So, Min $M(1,1) + M(2,3) + d_0 d_1 d_3$ $= 840$	

i	j	k	Condition	Value
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		2	$1 \leq 2 < 3$, So, $\text{Min}(M(1,2) + M(3,3) + d_1 d_2 d_3)$ $= 264$	
--	--	---	---	--

264

		3	$1 \leq 3 \leq 3$, \rightarrow False	
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We have to select Minimum value for $M[1, 3] = 264$

2	4	2	$2 \leq 2 < 4$, So, $\text{Min}(M(2,2) + M(3,4) + d_1 d_2 d_4)$ $= 1320$	
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		3	$2 \leq 3 < 4$, So, $\text{Min}(M(2,3) + M(4,4) + d_1 d_3 d_4)$ $= 2760$	1320
--	--	---	--	------

		4	$2 \leq 4 < 4$ \rightarrow False	
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We have to select Minimum value for $M[2, 4]$

3	5	3	$3 \leq 3 < 5$ \rightarrow $\text{Min}(M(3,3) + M(4,5) + d_2 d_3 d_5)$ $= 1932$	
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		4	$3 \leq 4 < 5$ \rightarrow $\text{Min}(M(3,4) + M(5,5) + d_2 d_4 d_5)$ $= 1140$	
--	--	---	--	--

i	j	k	Condition	Value
		5	$3 \leq 5 < 5 \rightarrow$ False	
We have to select Minimum Value for $M[3, 5]$				1140
1	4	1	$1 \leq 1 < 4 \rightarrow$ So, $\text{Min}(M[1, 1] + M[2, 4] + d_1 d_1 d_4)$ $= 2120$	
		2	$1 \leq 2 < 4 \rightarrow$ So, $\text{Min}(M[1, 2] + M[3, 4] + d_1 d_2 d_4)$ $= 1080$	1080
		3	$1 \leq 3 < 4 \rightarrow$ So, $\text{Min}(M[1, 3] + M[4, 4] + d_1 d_3 d_4)$ $= 1224$	
		4	$1 \leq 4 < 4 \rightarrow$ False	

We have to select Minimum value for $M[1, 4]$

2	5	2	$2 \leq 2 < 5 \rightarrow$ $\text{Min}(M[2, 2] + M[3, 5] + d_1 d_2 d_5)$ $= 1350$
		3	$2 \leq 3 < 5 \rightarrow$ $\text{Min}(M[2, 3] + M[4, 5] + d_1 d_3 d_5)$ $= 2880$

i	j	k	Condition	Value
		4	$2 < 4 < 4 \rightarrow$ So, $\text{Min}(M(2, 4) + M(5, 5) + d_1 d_4 d_5)$ $= 2720$	1350

We have to select Minimum value for $M(2, 3)$

1	5	1	$1 < 1 < 5 \rightarrow$ So, $\text{Min}(M(1, 1) + M(2, 5) + d_1 d_1 d_5)$ $= 1630$	
		2	$1 < 2 < 5 \rightarrow$ So, $\text{Min}(M(1, 2) + M(3, 5) + d_1 d_2 d_5)$ $= 1344$	1344
		3	$1 < 3 < 5 \rightarrow$ So, $\text{Min}(M(1, 3) + M(4, 5) + d_1 d_3 d_5)$ $= 2280$	
		4	$1 < 4 < 5 \rightarrow$ So, $\text{Min}(M(1, 4) + M(5, 5) + d_1 d_4 d_5)$ $= 1640$	
		5	$1 < 5 < 5 \rightarrow$ False	

We have to select Minimum value for $M(1, 5)$

From the table, Top of the Table ~~corn~~ corner.

For Find the 5 Matrix Multiplications, we require 1344 Multiplication.

For $k=2$ we get 1344 value.

So, we have to divided Matrix into ~~two~~ Part, upto Two Matrix

$$M[1, 5] = ([A_1] \times [A_2]) ([A_3] \times [A_4] \times [A_5])$$

After that we have to take Previous value of 1344.

So, value is 1080.

For $k=4$, we get 1080 value.

So, we have to divided matrix upto 4 Matrix.

$$M[1, 5] = ([A_1] \times [A_2]) ([A_3] \times [A_4]) \times [A_5]$$

* Explain Longest Common Subsequence with Dynamic Programming.

=> For Finding the Longest Common Subsequence, We have to follow this method.

Step:

- 1 IF String A Length is n and String B Length is m than create the $(n+1) \times (m+1)$ table which index is start with 0.
- 2 So, String A_i will start with 0 to n and String B_j will start with 0 to m in table
- 3 We have to Find the table value using this Formula.

Row-wise we have to Find value.

$$C[i, j] = \begin{cases} 0, & \text{if } i=0 \text{ or } j=0 \\ C[i-1, j-1] + 1, & \text{if } i > 0 \text{ and } j > 0 \text{ and } a_i = b_j \\ \max(C[i, j-1], C[i-1, j]), & \text{if } i > 0 \text{ and } j > 0 \text{ and } a_i \neq b_j \end{cases}$$

4 According to $C[i, j]$ condition we have to put sign.

5 Bottom of the table we get answer.

If we get n answer than we have to select n string or character.

6 We have to follow the sign. When we get \wedge sign than we have to take this character.

Ex. String $A = ABCB$ and $B = BDCAB$.

→ Here, Length of String A is 4 and Length of string B is 5.

So, we have to create table for $A \times B$ in which A is start with 0 to 4 and B is start with 0 to 5.

After that we have to find value of $C[i, j]$.

		0	1	2	3	4	5
		B _j	B	D	C	A	B
0	A _i	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	<u>1</u>
2	B	0	<u>1</u>	<u>1</u>	<u>1</u>	1↑	2↖
3	C	0	1↑	2↑	2↑	<u>2</u>	2↑
4	D	0	1↑	2↑	2↑	2↑	3↖

For $i = 0$ or $j = 0$, we have to take $C[i, 0] = C[0, j] = 0$

i	j	Condition	Value
1	1	$a_1 = b_1 = \max(C[i, 0], C[0, j])$ $= \max(0, 0) = 0↑$	0↑
1	2	$a_1 \neq b_2 = \max(C[i, 1], C[0, 2])$ $= \max(0, 0) = 0↑$	0↑
1	3	$a_1 \neq b_3 = \max(C[i, 2], C[0, 3])$ $= \max(0, 0) = 0↑$	0↑
1	4	$a_1 = b_4 = A = A = (C[0, 3] + 1)$ $= (0 + 1) = 1↖$	1↖

i	j	Condition	Value
1	5	$a_1 = b_5, A \neq B,$ $\text{Max}(C[1,4], C[0,5]) = \text{Max}(1, 0)$ $= 1 \leftarrow$	$1 \leftarrow$
2	1	$a_2 = b_1, B = B,$ $(C[1,0] + 1) = 1 \curvearrowright$	$1 \curvearrowright$
2	2	$a_2 \neq b_2, B \neq D$ $\text{Max}(C[2,1], C[1,2])$ $= \text{Max}(1, 0) = 1 \leftarrow$	$1 \leftarrow$
2	3	$a_2 \neq b_3, B \neq C,$ $\text{Max}(C[2,2], C[1,3])$ $= \text{Max}(1, 0) = 1 \leftarrow$	$1 \leftarrow$
2	4	$a_2 \neq b_4, B \neq A$ $= \text{Max}(C[2,3], C[1,4])$ $= \text{Max}(1, 1) = 1 \uparrow$	$1 \uparrow$
2	5	$a_2 \neq b_5, B = B$ $= (C[1,4] + 1) = 1 + 1 = 2 \curvearrowright$	$2 \curvearrowright$
3	1	$a_3 \neq b_1, C \neq B$ $= \text{Max}(C[3,0], C[2,1])$ $= \text{Max}(0, 1) = 1 \uparrow$	$1 \uparrow$
3	2	$a_3 \neq b_2, C \neq D$ $= \text{Max}(C[3,1], C[2,2])$ $= \text{Max}(1, 1) = 1 \uparrow$	$1 \uparrow$

i	j	Condition	Value
3	3	$a_3 = b_3, C = C$ $C[2,2] + 1 = 1 + 1 = 2 \uparrow$	$2 \curvearrowright$
3	4	$a_3 \neq b_4, C \neq A$ $= \text{Max}(C[3,3], C[2,4])$ $= \text{Max}(2, 1) = 2 \leftarrow$	$2 \leftarrow$
3	5	$a_3 \neq b_5, C \neq B$ $= \text{Max}(C[3,4], C[2,5])$ $= \text{Max}(2, 2) = 2 \uparrow$	$2 \uparrow$
4	1	$a_4 = b_1, B = B$ $= C[3,0] + 1 = 0 + 1 = 1 \uparrow$	$1 \curvearrowright$
4	2	$a_4 \neq b_2, B \neq D$ $= \text{Max}(C[4,1], C[3,2])$ $= \text{Max}(1, 1) = 1 \uparrow$	$1 \uparrow$
4	3	$a_4 \neq b_3, B \neq C$ $= \text{Max}(C[4,2], C[3,3])$ $= \text{Max}(1, 2) = 2 \uparrow$	$2 \uparrow$
4	4	$a_4 \neq b_4, B \neq A$ $= \text{Max}(C[4,3], C[3,4])$ $= \text{Max}(2, 2) = 2 \uparrow$	$2 \uparrow$
4	5	$a_4 = b_5, B = B$ $= C[3,4] + 1 = 2 + 1 = 3 \uparrow$	$3 \curvearrowright$

Bottom of the table, we get the answer.

Here, we get 3 value. So, we get string with length 3.

Where we get \wedge sign, this character we have to select.

So, string become BCB.

* Explain 0-1 knapsack Problem with Dynamic Programming.

\Rightarrow 0-1 knapsack Problem is used to select n different things according to its weight.

Using this step, we can solve the 0-1 knapsack Problem.

Step :

1 Take Number of Item as a and take Maximum Weight as M .

IF Number of Item is n and Maximum weight is m than

We have to create table for i and M .

2 Item index is started with 0 to n and Weight index is started with 0 to m .

3 Using this Condition, We have to Find table values.

$$C[i, M] = \begin{cases} 0 & , \text{ if } i = 0 \text{ or } M = 0 \\ C[i-1, M], w_i > M \\ \max(V_i + C[i-1, M-w_i], C[i-1, M]) & , w_i < M \end{cases}$$

Here, w_i = Weight of i th item
 V_i = Value of i th item

4 Bottom of the table, we get the Maximum profit value.

5 For select the item, we have to use this formula.

if $C[i, M] \neq C[i-1, M]$
 $i = i - 1$
 $M = M - w_i$
else $i = i - 1$

Ex.	Item	1	2	3	4
	Weight	2	3	4	5
	Value	3	4	5	6

Maximum Weight = 5 Kg

Here, There are Four item and Maximum weight 5.

So, We have to create table, For $i = 0$ to 4 and $M = 0$ to 5.

i \ m	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	3	3	3	3
2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0					

Using the formula, We have to find the value.

We have to find ~~row~~ row-wise value.

For $i = 0$ or $M = 0$, $C[0, M] = C[M, 0] = 0$.

i	j	Condition	Value
1	1	$w_1 > 1, 2 > 1$ $C[1, 1] = C[0, 1] = 0$	0
1	2	$w_1 = 2, 2 = 2$ $\text{Max}(3 + C[0, 0], C[0, 2])$ $= \text{Max}(3, 0) = 3$	3
1	3	$w_1 < 3, 2 < 3$ $= \text{Max}(3 + C[0, 1], C[0, 3])$ $= \text{Max}(3, 0) = 3$	3
1	4	$w_1 < 4, 2 < 4$ $= \text{Max}(3 + C[0, 2], C[0, 4])$ $= \text{Max}(3, 0) = 3$	3
1	5	$w_1 < 5, 2 < 5$ $= \text{Max}(3 + C[0, 3], C[0, 5])$ $= \text{Max}(3, 0) = 3$	3
2	1	$w_2 > 1, 3 > 1$ $= C[2, 1] = C[1, 1] = 0$	0
2	2	$w_2 > 2, 3 > 2$ $= C[2-1, 2] = C[1, 2] = 3$	3

i	j	Condition	Value
2	3	$w_2 = 3, 3 = 3$ $= \text{Max}(4 + C[0,0], C[1,3])$ $= \text{Max}(4, 3) = 4$	4
2	4	$w_2 < 4, 3 < 4$ $= \text{Max}(4 + C[0,1], C[1,4])$ $= \text{Max}(4, 3) = 4$	4
2	5	$w_2 < 5, 3 < 5$ $= \text{Max}(4 + C[1,2], C[1,5])$ $= \text{Max}(7, 3) = 7$	7
3	1	$w_3 > 1, 4 > 1$ $= C[3-2, 1] = C[2, 1] = 0$	0
3	2	$w_3 > 2, 4 > 2$ $= C[3-2, 2] = C[2, 2] = 3$	3
3	3	$w_3 > 3, 4 > 3$ $= C[3-2, 3] = C[2, 3] = 4$	4
3	4	$w_3 = 4, 4 = 4$ $= \text{Max}(5 + C[1,0], C[2,4])$ $= \text{Max}(5, 4) = 5$	5
3	5	$w_3 < 5, 4 < 5$ $= \text{Max}(5 + C[2,1], C[2,5])$ $= \text{Max}(5, 7) = 7$	7

i	j	Condition	Value
4	1	$W_4 > 1, S > 1$ $C[4-1, 1] = C[3, 1] = 0$	0
4	2	$W_4 > 2, S > 2$ $C[4-1, 2] = C[3, 2] = 3$	3
4	3	$W_4 > 3, S > 3$ $C[4-1, 3] = C[3, 3] = 4$	4
4	4	$W_4 > 4, S > 4$ $C[4-1, 4] = C[3, 4] = 5$	5
4	5	$W_5 = 5, S = 5$ $= \text{Max}(C_5 + C[3, 0], C[3, 5])$ $= \text{Max}(5, 7) = 7$	7

For select the item,

Bottom of the table, we get 7 value, which is maximum Profit value at $i=4$ and $m=5$

For $i=4$ and $m=5$

$$C[i, m] = C[4, 5] = 7$$

$$C[i-1, m] = C[3, 5] = 7$$

$$\text{So, } i = i-1 = 4-1 = 3$$

For $i = 3$ and $m = 5$

$$C[i, m] = C[3, 5] = 7$$

$$C[i-1, m] = C[2, 5] = 7$$

$$\therefore i = i - 1 = 3 - 1 = 2$$

For $i = 2$ and $m = 5$

$$C[i, m] = C[2, 5] = 7$$

$$C[i-1, m] = C[1, 5] = 3$$

$$\text{So, } m = 5 - w_2 = 5 - 3 = 2$$

$$i = i - 1 = 2 - 1 = 1$$

For $i = 1$ and $m = 2$

$$C[i, m] = C[1, 2] = 3$$

$$C[i-1, m] = C[0, 2] = 0$$

$$\text{So, } m = 2 - w_1 = 2 - 2 = 0$$

$$i = i = i - 1 = 0$$

So, Selected Item is Item 1
and Item 2.

* Explain Making Change Problem.

=> This are the Basic steps to Perform Making Change Problem.

Step:

1 If We have D Coin mean d value coins and we have to create n value For making change.

2 Create a table for $D \times n$ with value of D start with 0 to d and value of n start with 0 to n

3 Using this formula, we can find the values of table,

$$C[i, j] = \begin{cases} 1 + C[i, j - d_1] & , i = 1 \\ C[i - 1, j] & , j < d_i \\ \min(C[i - 1, j], 1 + C[i, j - d_i]) & , \text{other} \end{cases}$$

4 Bottom of the table we get minimum number of coin.

5 For Finding the Coin value,

if $C[i, j] \neq C[i-1, j]$

$j = j - d_i$

~~$i = i - 1$~~

else

$i = i - 1$

Ex. Make a table of number of demonization $(1, 4, 6)$ and amount (8) .

=> Here, we have three coin and $N = 8$

So, we have to create table for d/n from 0 to 3, to 0 to 8.

i	D	j								
		0	1	2	3	4	5	6	7	8
0	-	0	0	0	0	0	0	0	0	0
1	1	0	1	2	3	4	5	6	7	8
2	4	0	1	2	3	1	2	3	4	2
3	6	0	1	2	3	1	2	1	2	2

For $C[i, 0] = C[0, j] = 0$ and
Using the formula, we have
to Find column-wise table value.

i	j	d_i	Condition	Value
1	1	1	$= 1 + C[1, 1-1] = 1 + C[1, 0] = 1$	1
2	1	4	$4 < 1, j < d_i, = C[2-1, 1]$ $= C[1, 1] = 1$	1
3	1	6	$j < d_3, = C[3-1, 1] = C[2, 1] = 1$	1
1	2	1	$= 1 + C[1, 2-1] = 1 + C[1, 1]$ $= 1 + 1 = 2$	2
2	2	4	$j < d_2, = C[2-1, 2] = C[1, 2] = 2$	2
3	2	6	$j < d_3 = C[3-1, 2] = C[2, 2] = 2$	2
1	3	1	$= 1 + C[1, 3-1] = 1 + C[1, 2] = 3$	3
2	3	4	$j < d_2, = C[2-1, 3] = C[1, 3] = 3$	3
3	3	6	$j < d_3 = C[3-1, 3] = C[2, 3] = 3$	3
1	4	1	$= 1 + C[1, 4-1] = 1 + C[1, 3] = 4$	4
2	4	4	$= \min(C[2-1, 4], 1 + C[2, 4-4])$ $= \min(C[1, 4], 1 + C[2, 0])$ $= \min(4, 1) = 1$	1

i	j	d _i	Condition	Value
3	4	6	$j < d_3, C[3-1, 4] = C[2, 4] = 1$	1
1	5	1	$= 1 + C[1, 5-1] = 1 + C[1, 4] = 5$	5
2	5	4	$\min(C[2-1, 5], 1 + C[2, 5-4])$ $= \min(C[1, 5], 1 + C[2, 1])$ $= \min(5, 2) = 2$	2
3	5	6	$j < d_3, C[3-1, 5] = C[2, 5] = 2$	2
1	6	1	$= 1 + C[1, 6-1] = 1 + C[1, 5] = 6$	6
2	6	4	$= \min(C[1, 6], 1 + C[2, 2])$ $= \min(6, 3) = 6$	6
3	6	6	$= \min(C[2, 6], 1 + C[3, 0])$ $= \min(6, 1)$	1
1	7	1	$= 1 + C[1, 7-1] = 1 + C[1, 6] = 7$	7
2	7	4	$= \min(C[1, 7], 1 + C[2, 3])$ $= \min(7, 4) = 4$	4
3	7	6	$= \min(C[2, 7], 1 + C[3, 1])$ $= \min(4, 2) = 2$	2
1	8	1	$= 1 + C[1, 8-1] = 1 + C[1, 7] = 8$	8

i	j	d _i	Condition	Value
2	8	4	$= \text{Min}(C[1, 8], 1 + C[2, 4])$ $= \text{Min}(8, 2) = 2$	2
3	8	6	$= \text{Min}(C[2, 8], 1 + C[3, 2])$ $= \text{Min}(2, 3) = 2$	2

From the table, Bottom of the table we get 2 value.

So, We required Minimum 2 Coin For Create 8 value.

For Finding the value of Coin,

$$C[i, j] = C[3, 8] = 2$$

$$C[i-1, j] = C[2, 8] = 2$$

$$\text{So, } i = i-1 = 3-1 = 2$$

For $i = 2$ and $j = 8$

$$\therefore C[i, j] = C[2, 8] = 2$$

$$\therefore C[i-1, j] = C[1, 8] = 8$$

So, we have to take $i=2$ value coin.

$$\text{So, } j = j - d_i = 8 - 4 = 4$$

For $i = 2$ and $j = 4$

$$\therefore C[i, j] = C[2, 4] = 1$$

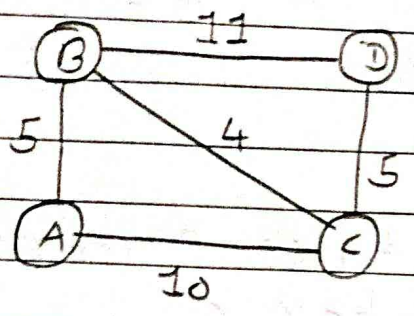
$$\therefore C[i-1, j] = C[1, 4] = 4$$

Again we have to take
 $i=2$ value coin.

$$\therefore j = j - d_2 = 4 - 4 = 0$$

For Make 8 Value we
require Two Four coins.

* Find the shortest Path Using Dijkstra's Algorithm.

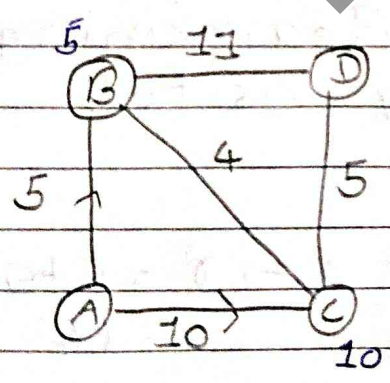


=> Using this step, we can find the Dijkstra's Algorithm.

We have to find minimum value for vertex path.

We have to find value for every vertex path.

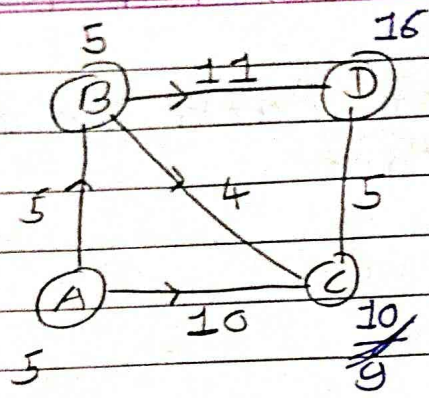
- Vertex: A: Vertex A is directly connected with B and C



From $A \rightarrow B = 5$
 $A \rightarrow C = 10$

Vertex B Value = 5
 Vertex C Value = ~~5~~ 10

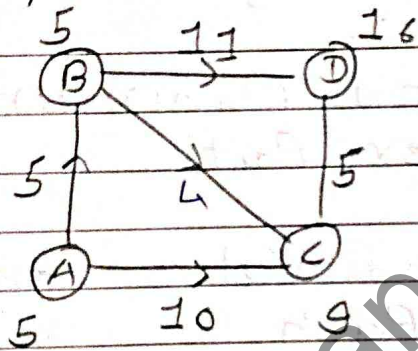
- Vertex B: Vertex B is directly connected with A, C and D But A is already visit



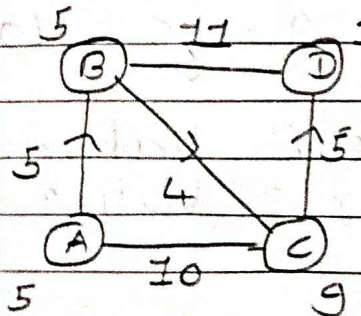
From $B \rightarrow D = 5 + 11 = 16$
 $B \rightarrow C = 5 + 4 = 9$

We have to take Minimum value for Vertex C.

So, Vertex C value = 9



- For Vertex C : For Vertex C we have to take only D vertex because Remaining vertex is already visited.



From $C \rightarrow D = 5 + 9 = 14$

For D we have to we have to take minimum value.

So, Vertex D Value = 14

Here, All the Vertex are visited.
So, Shortest Path weight will be 14.

Finding the Path we have to do backtracking with minimum value.

From D to C we get Minimum value 5

From C to B we get Minimum value 4.

From B to A we get Minimum value 5.

So, Path will be $A \rightarrow B \rightarrow C \rightarrow D$

