Unit 8: Introduction To Complexity Theory

* Define Class P and Class NP.

=> Class P :

The P Class stands for Polynomial Time.

P Class consistes those problem which require to slove the Polynomial Time.

P Class consists those problem which require $O(n^k)$ to slove time.

where K = Any Constant.

Ex. All the Searching and Sorting Algorithm.

-> Class NP :

The NP class stands for Non-

Deterministic Polynomial Time.

NP class consists those problem which is verifiable by some one.

NP class consists those problem which require to solve the different time.

NP Class Problem time can be change every time.

Decision Making Problem is include in NP class.

In NP Class, If there is same n input then every time this Problem gives different output.

* Define Polynomial Reduction:

=> If There is Two Problem P_1 and P_2 and Both Problem has its input.

Here, Problem P_1 has I_1 input and For solve the Problem algorithm is required.

But Problem P_1 does not have any type of algorithm.

Problem P_1 → Input (I_1)
Problem P_1 → Algorithm (Unknown)

Here, Problem P_2 has Input I_2 and For solve the Problem algorithm is required.

So, Problem P_2 does have algorithm For solve the problem.

Problem P_2 → Input (I_2)
Problem P_2 → Algorithm (Known)

If Problem P_1 is solve using the Problem P_2 input and algorithm so, It is called Polynomial Reduction.

So, Problem P_1 is Reduce by the Problem P_2.

* Define NP Hard Problem and NP Complete Problem.

=) NP Hard Problem:

If Every NP Problem is reduce by any P Problem So, this Problem is called NP Hard Problem.

Ex.
Problem P_1 is NP Problem and Problem P_2 has P Problem with its algorithm.

So Problem P_1 is Reduce by the P Problem.

So, this Problem P_1 is Called NP Hard Problem.

=) NP Complete Problem:

If Problem n is NP Problem and Every n' problem can ~~by~~ reduce ~~using~~ the Problem n. So this Problem is called NP Complete Problem.

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Ex.

Problem P1 is a NP Problem.
Problem P2 is also NP Problem.

If Problem P2 is Polynomial Reduce to Problem P1.

So This Problem P1 is called NP Complete Problem.

* Explain Traveling Salesman Problem Using Branch-and-Bound Method.

=> Traveling Salesman Problem :

Salesman have to travel from one destination to another destination.

Salesman have to find the shortest distance for the travel.

So, This Problem is called Traveling Salesman Problem

Ex.

$$A = \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$

=) Step 1: Select Minimum cost From Row-wise.

$$\begin{array}{cl} \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} & \begin{array}{l} \rightarrow 10 \\ \rightarrow 2 \\ \rightarrow 2 \\ \rightarrow 3 \\ \rightarrow 4 \end{array} \end{array}$$

Subtract From the matrix.

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

Step 2: Select Minimum cost From Column-wise.

$$\begin{array}{c} \begin{array}{ccccc} 1 & 0 & 3 & 0 & 0 \end{array} \\ \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \end{array}$$

Subtract From the Matrix :

$$\begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

$$A_{Reduce} = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix} \end{array}$$

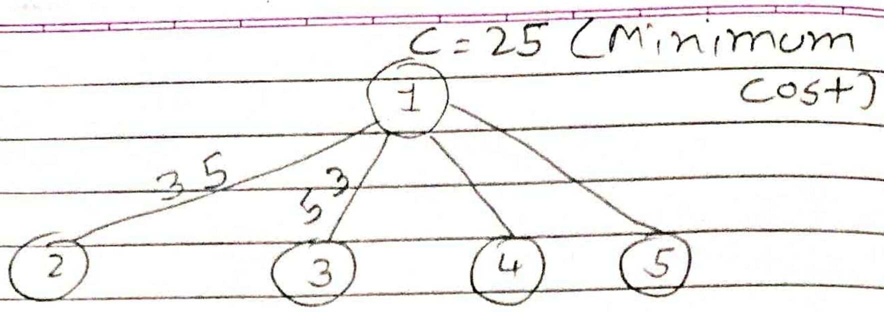
Here, Total Reduce Cost = Row-wise Cost + Column-wise Cost

= (10+2+2+3+4) + (1+0+3)

= 25

Here, Minimum cost is 25 or Maximum Cost 25.

So, We have to Find every Branch or node using state Tree.



We have to create, Reduce Matrix for every Node (2, 3, 4, 5)

→ For Node 2 : Reduce Matrix :

We have to ~~take~~ find cost from 1 to 2 So, we have to take Row 1 and column as a ∞ in A reduce Matrix.

$$A(2)\ \text{Reduce} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \begin{matrix} \to 0 \\ \to 0 \\ \to 0 \\ \to 0 \\ \to 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Here, Reduce cost is 0.

So, Cost from 1 to 2 = C(1, 2) + Minimum cost + Reduce cost

(C(1,2) value is take in the A reduce Matrix)

= 10 + 25 + 0

= 35

→ For Node 3 : Reduce Matrix :

Take column 3 and Row 1 as ∞ and Find Reduce cost.

A(3) Reduce

$$
= \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}
\begin{matrix} \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}
$$

$$
\begin{matrix} 11 & 0 & & 0 & 0 \end{matrix}
$$

$$
= \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}
$$

Reduce Cost = 11

$$
\therefore C(1,3) = C(1,3) + 25 + 11
$$
$$
= 17 + 25 + 11
$$
$$
= 53
$$

According Two Both the Node 2 and 3 we have to Find value of Node 4 and 5.

$$\Rightarrow A_{(4)} \text{ Reduse} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \quad R.C = 0$$

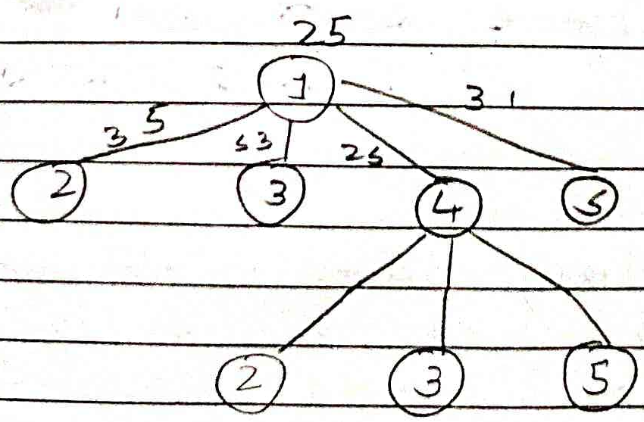
$$C(1,4) = C(1,4) + 25 + 0$$
$$= 25$$

$$\Rightarrow A(5) \text{ Reduse} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

$$C(1,5) = 31$$

Here, Minimum Cost of Node 1 to 5 is 25 which is cost of Node 4.

So, We have to select Node 4 and exhand the state tree.



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Again we have to Find cost of every Node From 4 to 2, 3 and 5.

We have to Use $A_{(4)}$ Reduse Matrix For Find the cost because 2, 3 and 5 are child of ~~Root~~ 4. Node

-> For Node 2 :

$$A_{(4-2)R.M} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$ Reduce Cost = 0

$C(4,2) = C(4,2) + C(4) +$ Redue cost

$= 3 + 25 + 0 = 28$

-> For Node 3 :

$$A_{(4-3)R.M} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

$C(4,3) = 50$

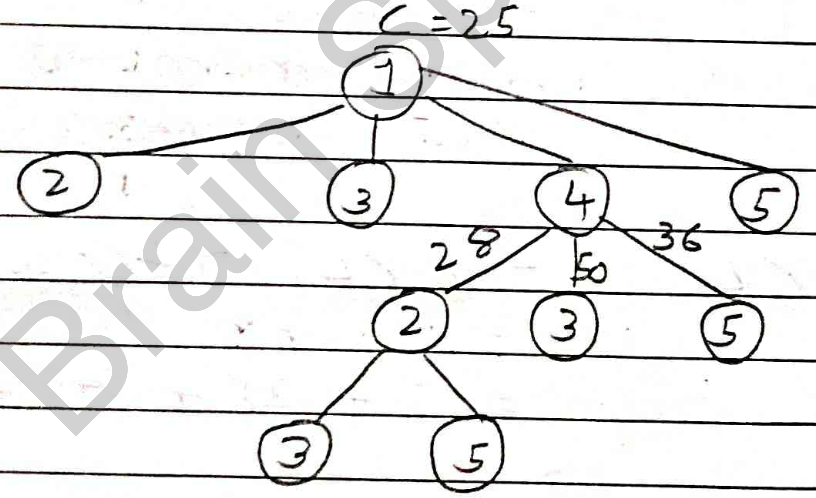
→ For Node 5:

$A_{(4-5)R.M.} =$

∞	∞	∞	∞	∞
1	∞	0	∞	∞
0	3	∞	∞	∞
∞	∞	∞	∞	∞
∞	0	0	∞	∞

$C(4,5) = 36$

Again, Node 4 to 6 Cost is minimum. So, we have to expand this Node.



Here, We have to use, Matrix (4-2) for Find the cost of Node 3 and 5.

→ For Node: 3 :

$$A_{(2-3)} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

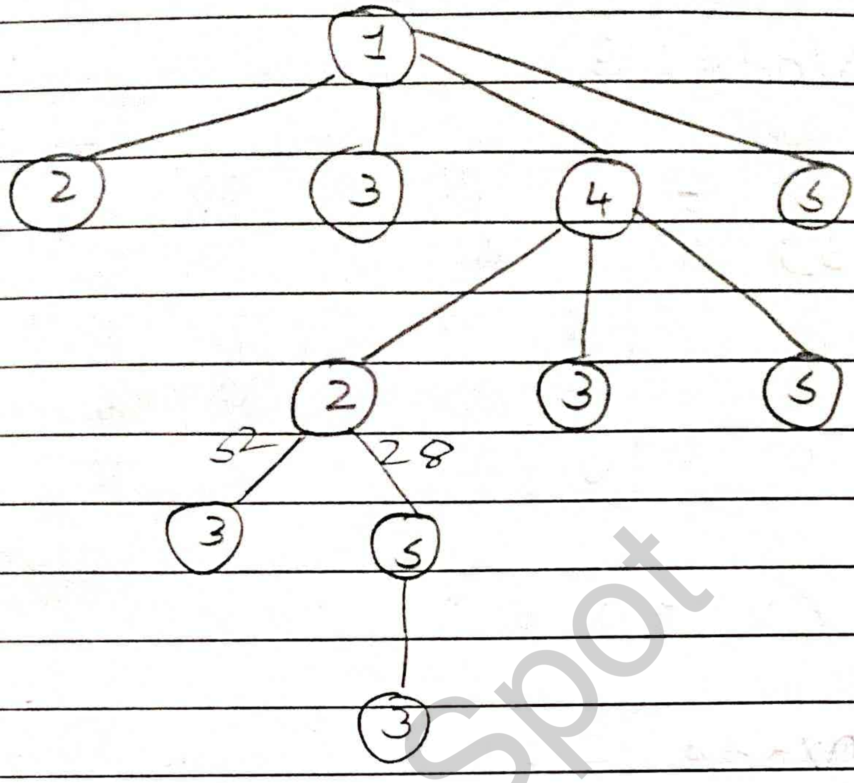
$C(2, 3) = 52$

→ For Node 5 :

$$A_{(2,5)} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

$C[2, 5] = 28$

Here, we can see that Node 2 to 5 has low cost so, we have to ~~exb~~ expand this Node.



$A_{(5-3)}$ =

Here, we reach the leaf Node of the state Tree with cost 28.

So, Minimum Cost is 28. and path is 1-4-2-5-3-1