Fuzzy Systems

* Explain Crisp Set with its Operation.

=) Crisp Set is also known as a Classical set or conventional set.

Boundaries of Crisp Set Function is Fixed and represent in binary either 0 or 1.

=) Crisp Set Operation:

1 Commutativity:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

2 Associativity:

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

3 Distributivity:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

4 Idenpotence:

$$A \cup A = A$$
$$A \cap A = A$$

5 Identity:

$A \cup \emptyset = A$

$A \cap \emptyset = \emptyset$

$A \cup E = E$

$A \cap E = A$

6 Law of Absorption:

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

7 Transitivity:

If $A \subseteq B$, $B \subseteq C$ then $A \subseteq C$

8 Involution: $(A')' = A$

9 Law of the exeludid middle:

$A \cup A' = E$

10 Law of Contradiction:

$A \cap A' = \emptyset$

11 De-Morgan's Law:

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

* Explain Fuzzy Set with its operation.

=> A Fuzzy Set is a generalization of a crisp set in the context of Fuzzy Logic.

Fuzzy Set deals with reasoning and decision-making.

Fuzzy Set allows For degrees of membership.

Membership Function element can belong to a certain degree between 0 and 1.

=> Fuzzy Set Operation:

1 $\mu_{(A \cup B)} = \max(\mu_A(x), \mu_B(x))$

2 $\mu_{(A \cap B)} = \min(\mu_A(x), \mu_B(x))$

3 $\mu_{(A')} = 1 - \mu_A(x)$

4 Dot Product:

$\mu_{(A \cdot B)} = \mu_A(x) \cdot \mu_B(x)$

5 Scaler Product

$$\mu_{\alpha A}(x) = \alpha \times \mu_A(x)$$

6 Equality (A = B):

$$\mu_A(x_1) = \mu_B(x_1)$$

7 Power of a Fuzzy Set

$$\mu_{A^\alpha}(x) = (\mu_A(x))^\alpha$$

8 Difference:

$$A - B = (A \cap B')$$

9 Disjuctive Sum:

$$A \oplus B = (A' \cap B) \cup (A \cap B')$$

10 Fuzzy Relation:

$$\mu_R(x, y) = \mu_{A \times B}(x, y)$$

$$= \min(\mu_A(x), \mu_B(x))$$

* Difference Between Crisp Set and Fuzzy Set.

=>	Crisp Set	Fuzzy Set
1	Crisp Set defines value either 0 and 1.	Fuzzy Set defines value between 0 and 1.
2	Shows Full Membership.	Shows Partial Membership.
3	Bi-valued Function Logic	Infinite valued Function Logic
4	Used For Digital Design.	Used in Fuzzy Controller.
5	Defined by Precise Characteristics	Defined by ambiguous Property.
6	Crisp Set is a Classical Set.	Fuzzy Set is an extension of Crisp Set.

* Give Example of Minmax Composition.

=> $$R = \begin{matrix} \\ x_1 \\ x_2 \end{matrix} \begin{matrix} \begin{matrix} z_1 & z_2 \end{matrix} \\ \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{matrix}$$

$$S = \begin{matrix} \begin{matrix} z_1 & z_2 & z_3 \end{matrix} \\ \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{matrix}$$

$\mu_T(x_1, z_1) = \max(\min(x_1, z_1), \min(x_1, z_1))$

$\mu_T(x_1, z_1) = \max(\min(0.6, 1), \min(0.3, 0.8))$

$= \max(0.6, 0.3)$

$= 0.6$

$\mu_T(x_1, z_2) = \max(\min(0.6, 0.5), \min(0.9, 0.4))$

$= \max(0.5, 0.3)$

$= 0.5$

$\mu_T(x_1, z_3) = \max(\min(0.6, 0.3), \min(0.3, 0.7))$

$= \max(0.3, 0.3)$

$= 0.3$

$\mu_T(x_2, z_1) = \max(\min(0.2, 1), \min(0.9, 0.8))$

$= \max(0.2, 0.8)$

$= 0.8$

$\mu_T(x_2, z_2) = \text{Max}(\min(0.2, 0.5), \min(0.9, 0.4))$

$= \text{Max}(0.2, 0.4)$

$= 0.4$

$\mu_T(x_2, z_3) = \text{Max}(\min(0.2, 0.3), \min(0.9, 0.7))$

$= \text{Max}(0.2, 0.7)$

$= 0.7$

$$T = R \cdot S = \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix}$$

* Explain Fuzzy Rule based System.
or
Explain Fuzzy IF-Then Rule.

=> Fuzzy "IF-then" rules are fundamental components of Fuzzy Logic and Fuzzy Rule based system.

This rules express relationships between input and output variables.

IF part is known as Antecedent and Then part is know as Consequent

IF Part describes the conditions

based on Fuzzy Set.

If part is involves one or more Fuzzy propositions using terms like "is", "are", "very" etc.

Then part is specifies the action to be taken based on the conditions.

If Antecedent Then Consequent.

Ex. If Temperature is high then Pressure is Low.

$$R : T_{high} \rightarrow P_{Low}$$

$$T_{high} = \{(2.5, 0.1), (30, 0.2), (35, 0.5), (40, 0.6)\}$$

$$P_{low} = \{(2, 0.3), (5, 0.5), (6, 0.4)\}$$

$$R_{T \times P} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.3 & 0.5 & 0.4 \\ 0.3 & 0.5 & 0.4 \end{bmatrix}$$

* Explain Fuzzy Decision Making with its Types.

=) Decision Making activity steps taken to select the alternatives From choosing the certain Goal.

Fuzzy Decision Making methods are useful in situation where crisp logic may not sufficient For capture complexity of the decision problem.

This methods allow For the representation of uncertainty of decision-making process.

=) Steps For Decision Making:

1 Determine set of the alternative

2 Evaluate the alternative

3 Compare between the alternative.

There are Four Types of Decision Making.

1) Individual DM
2) Multiperson DM
3) Multiobjective DM
4) Multi-Attribute DM

1 Individual Decision Making:

In this Decision Making, Only a One Single Person is resposible For taking the decision.

This Model's characteristics as:

- Set of all Possible actions.
- set of Goal $G_i (i \in X_n)$
- Set of Constraints $C_j (j \in X_n)$
- $G_i(a)$ = Composition $G_i(a)$
- $C_j(a)$ - Composition $C_j(a)$

$$F_D = \text{Min}(i \in X_n F(G_i(a)), j \in X_n F(G_j(a)))$$

2 Multiperson Decision Making:

In this Decision Making, Multiple Person is resposible For taking the decision.

Here, Number of Person = $NC(x_i, x_j)$
Total No. of DM = n

then $SC(x_i, x_j) = \frac{NC(x_i, x_j)}{n}$

3 MultiObjective DM:

In this Decision Making, many objective are used for the taking decision.

There are Two Problem:

- To Acquired Proper Information related objectives by different alternative.
- To wait relative importans of each objectives.

Let a Universe of n alternative as

$A = \{a_1, a_2, ---, a_n\}$

set of m objectives as

$F = \{O_1, O_2, ---, O_n\}$

Decision $F^n = O_1 \cap O_2 --- \cap O_n$

$$m_F(a^*) = \max_{a \in A} [\mu_{DF}(a)]$$

4 Multi-Attribute Decision Making:

In this Decision Making, Evaluation of alternative can be carried out based on serval attribute ~~of~~ of object.

This Attribute can be numerical data.

$$Y = A_1X_1 + A_2X_2 + \cdots + A_nX_n$$

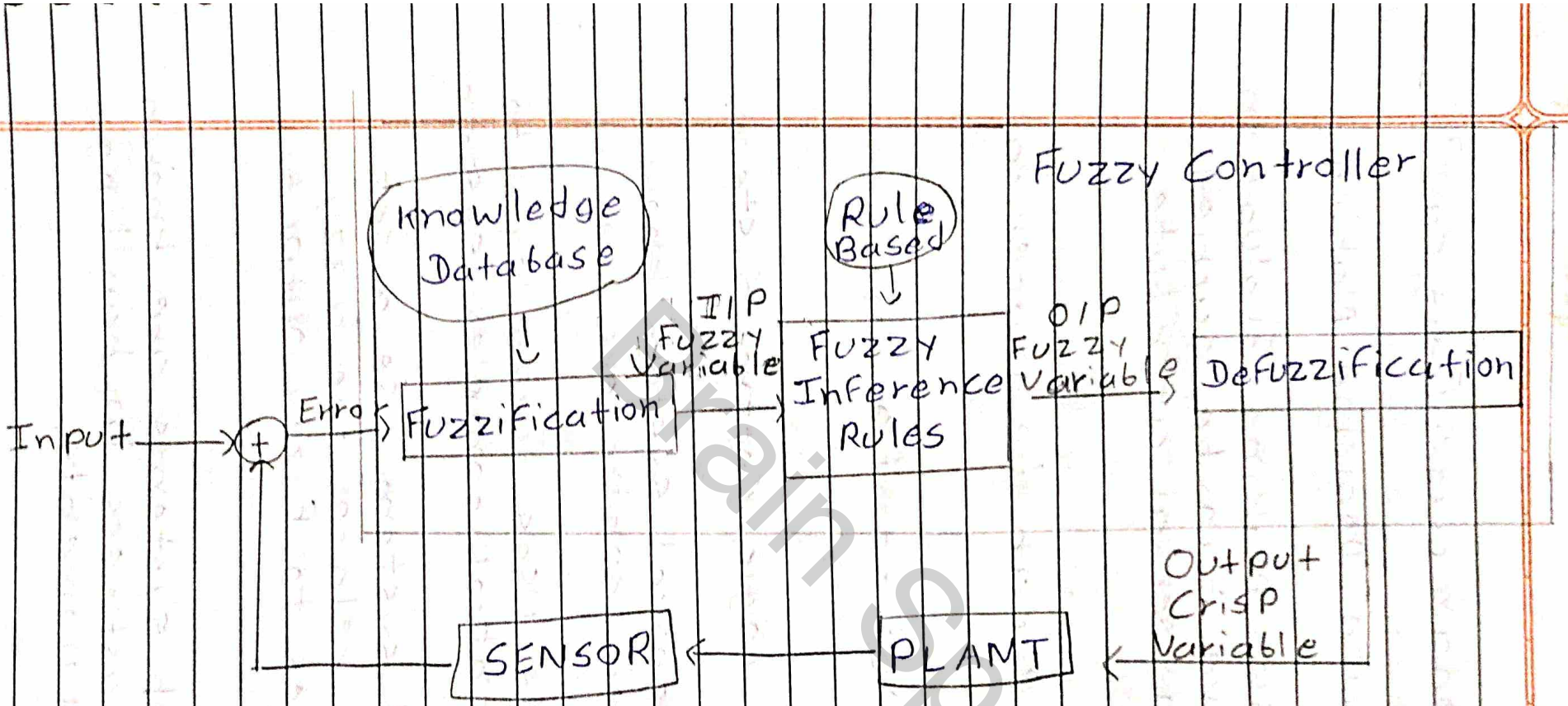
* Explain Fuzzy Logic Control System.

=> Fuzzy Logic Control system know as Fuzzy Logic controllers.

This system is uses Fuzzy logic to model and control complex system.

Fuzzy Control system use a set of rules that relate the fuzzy input variables to Fuzzy output variable.

Fuzzy Control systems are known for their Robustness in handling uncertain enviroments.


Fuzzy Controller
Knowledge Database
Rule Based
I/P Fuzzy Variable
O/P Fuzzy Variable
Input
Error
Fuzzification
Fuzzy Inference Rules
Defuzzification
Output Crisp Variable
SENSOR
PLANT

There are Five Major Components of Controller System.

1) Fuzzyfication
2) Fuzzy knowledge Base
3) Fuzzy Rule Base
4) Inferene Engine
5) DeFuzzyfication

1 Fuzzyfication: It is used to convert crisp Input values into the Fuzzy values.

2 Fuzzy Knowledge Base: It is used to normalize the parameter and find solution of Input and output of Membership Function.

3 Fuzzy Rule Base: It is used to selection of Fuzzy control rules with Input variable.

4 Inferene Engine: It is most important part of the Fuzzy Logic Control System.

It is used to Simulate human decision by performing the approximate reasoning.

5 DeFuzziFication: It is used to convert Fuzzy values into the Crisp value.

=> Advantages:

Fuzzy Logic Controlt System creation is cheaper and Robust compare to other controlt System.

Fuzzy Logic Controller is more Reliable and EFFicience compare to other system.

=> Disadvantages:

This system requires lots of data and high human expertise.

7 Define two Fuzzy sets I and F.

I = {(E, 0.5), (E, 0.2), (X, 0.1), (Y, 0.1), (I, 0.9), (T, 0.9)}

F = {(F, 0.99), (E, 0.6), (X, 0.1), (Y, 0.2), (I, 0.5), (T, 0.5)} Find the Following,

(a) I - F

I - F = {(F, -0.49), (E, -0.4), (X, 0), (Y, -0.1), (I, 0.4), (T, 0.4)}

(b) F ∪ F'

F' = {(F, 0.01), (E, 0.4), (X, 0.9), (Y, 0.8), (I, 0.5), (T, 0.5)}

F ∪ F' = {(F, 0.5), (E, 0.4), (X, 0.9), (Y, 0.8), (I, 0.9), (T, 0.9)}

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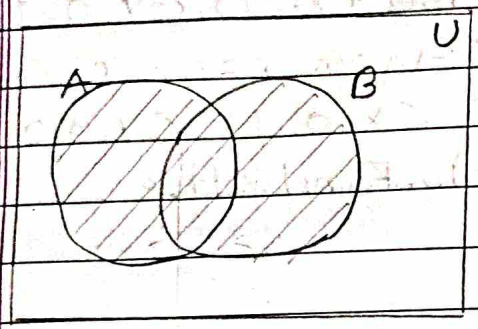
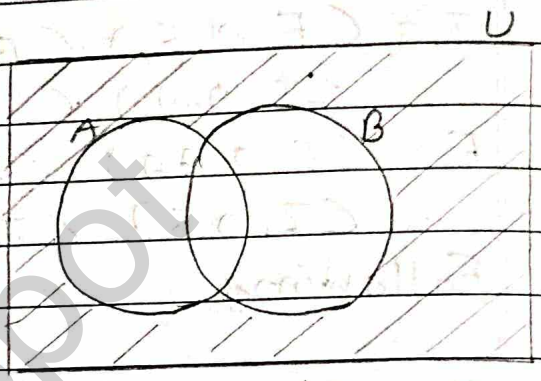
8 Explain De Morgan's Law

=> $(A \cup B)' = A' \cap B'$

-> L.H.S. $= (A \cup B)' =$

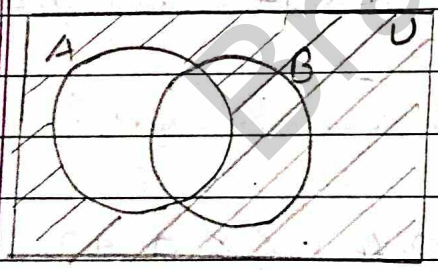


$A \cup B$

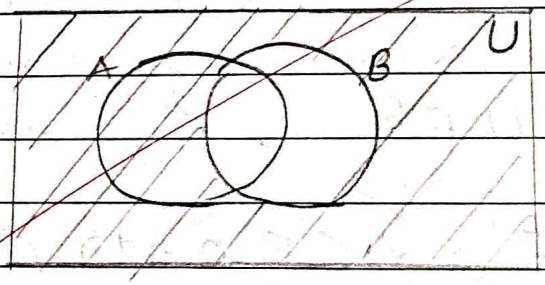


$(A \cup B)'$

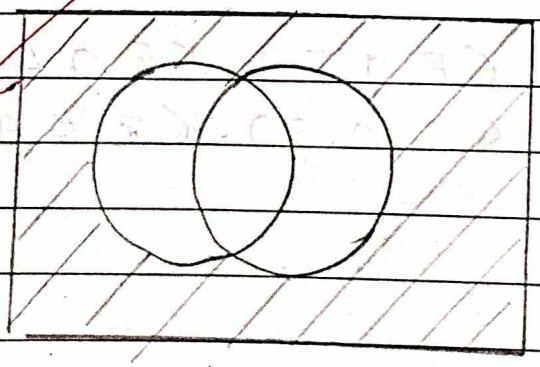
- R.H.S. $= A' \cap B'$



A'



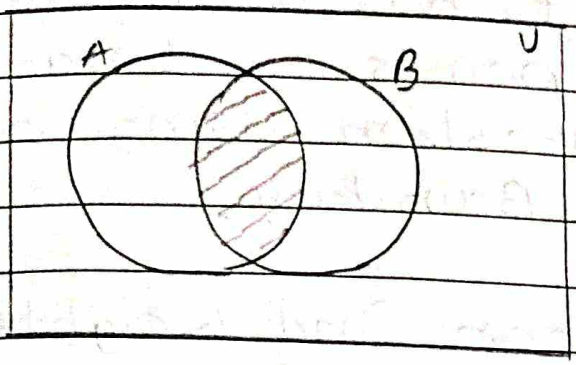
B'



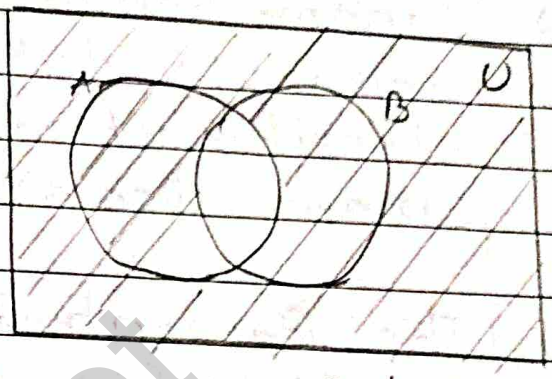
$(A' \cap B')$

⇒ $(A \cap B)' = A' \cup B'$

→ L.H.S. $= (A \cap B)'$

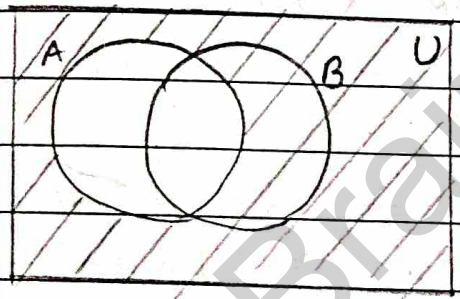


$A \cap B$

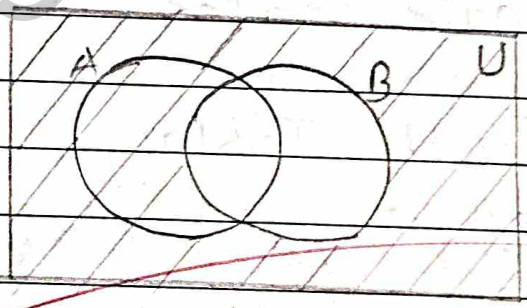


$(A \cap B)'$

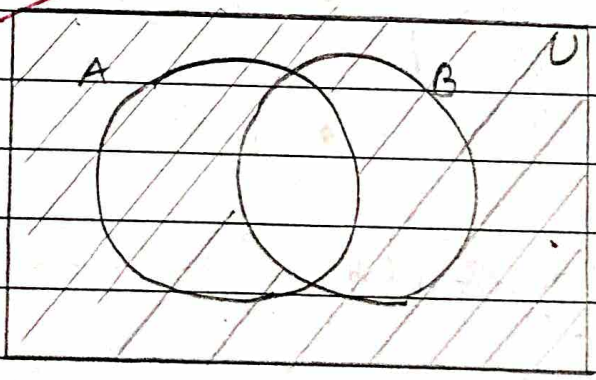
→ R.H.S. $= A' \cup B'$



A'



B'



$A' \cup B'$

Q Explain Defuzzification Methods.

=> DeFuzzification is used to convert Fuzzy set into a Crisp Set.

There are Four method of Defuzzification method.

a) Lamda Cut Method
b) Maxima Method
c) Weighted Average Method
d) Centroied Method.

a Lamda Cut Method:

Fuzzy Set(A) → Crisp Set(A)

$A\lambda = \{x / \mu_A(x) \geq \lambda\}$,

$0 \leq \lambda \leq 1$

Ex. $A = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6)\}$

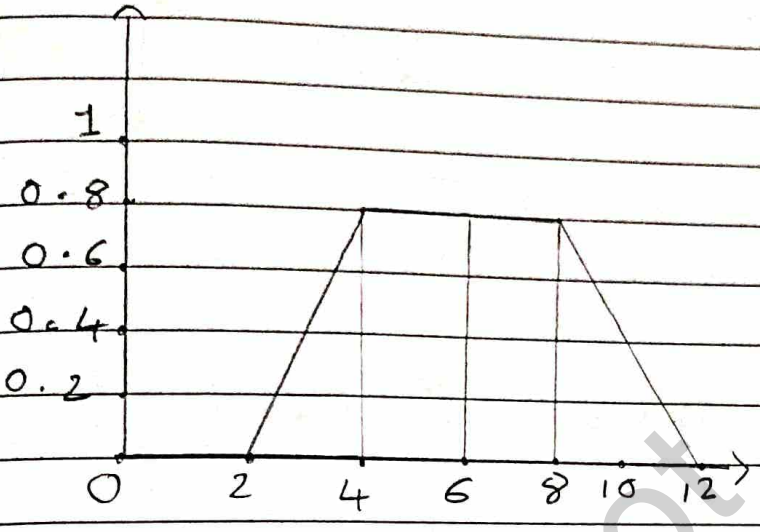
$\lambda = 0.3$

Crisp Set $= A_{0.3} = \{x_2, x_3\}$

Here, $x_2 \rightarrow 0.4 > 0.3$

$x_3 \rightarrow 0.6 > 0.3$

b Maxima Method:



(a) First of Maxima $(x^*) = 4$

(b) Last of Maxima $(x^*) = 8$

(c) Mean of Maxima

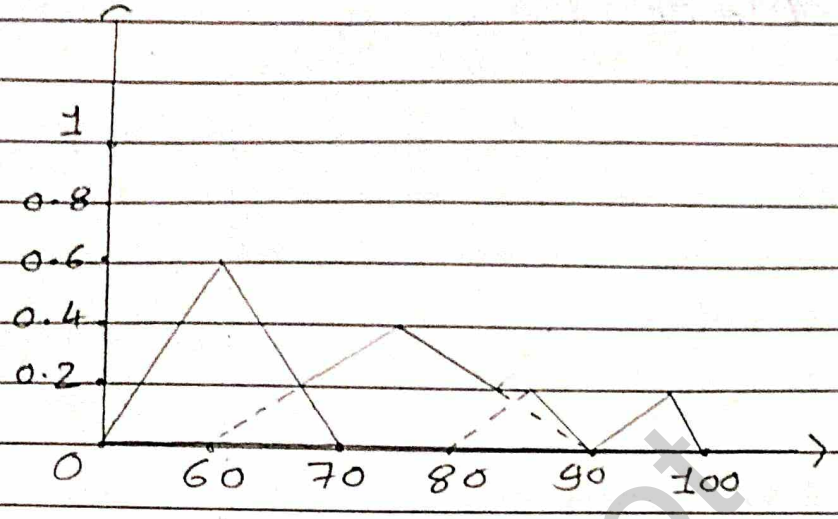
$$x^* = \frac{\sum x_i \in M x_i}{|M|}$$

$$= \frac{6+4+8}{3}$$

$$x^* = 6$$

c Weighted Average Method.

$$x^* = \frac{\sum \mu_x \, x}{\sum \mu(x)}$$

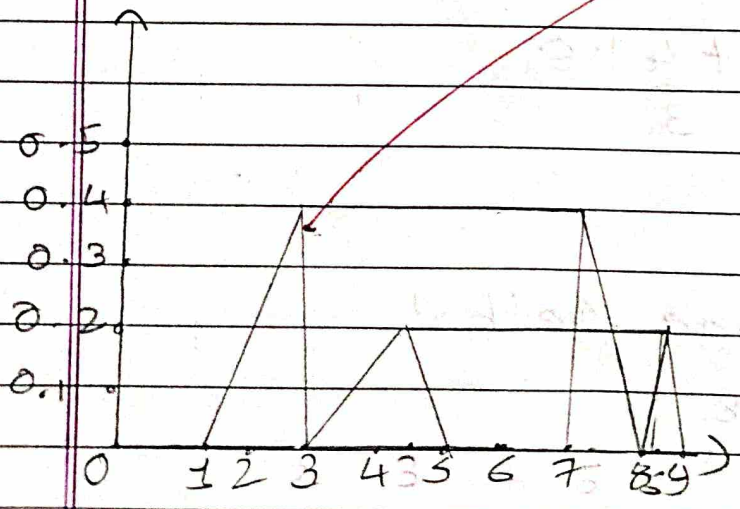


$$X^* = \frac{60 \times 0.6 + 70 \times 0.4 + 80 \times 0.2 + 90 \times 0.2}{0.6 + 0.4 + 0.2 + 0.2}$$

$$X^* = 70$$

d Centroied Method

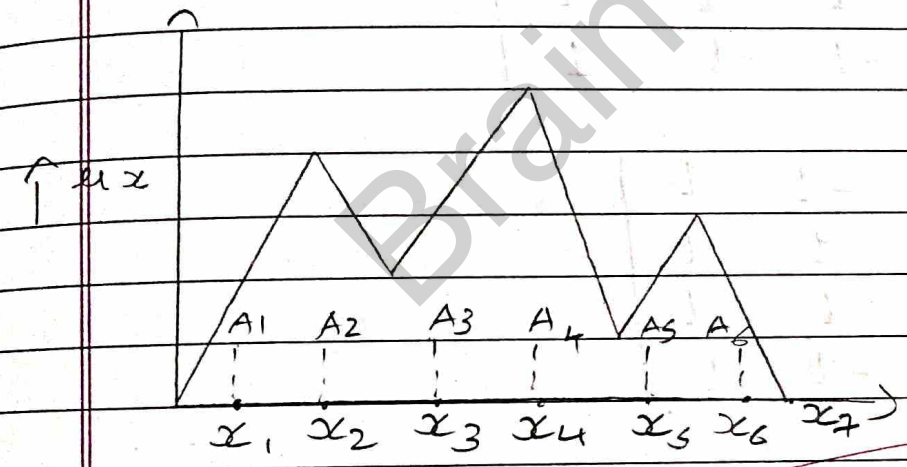
(1) Center of Sum



$$X^* = \frac{\Sigma A x_c}{\Sigma A}$$

Area = $\frac{1}{2}$ (Addition of 2 Base × height)

(2) Center of Gravity:



$$X^* = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

3 P = Mary is efficient, T(P) = 0.8
Q = Ram is efficient, T(Q) = 0.65,
Use the given Fuzzy statements to Find out Fuzzy truth value.
a) Mary is not efficient
b) Mary is efficient and so is Ram.
c) Either Mary or Ram efficient
d) If Mary is efficient then so is Ram.

=> Given, P = Mary is efficient
Q = Ram is efficient

a P' : Mary is not efficient
$T(P') = 1 - T(P)$
$= 1 - 0.8$
$= 0.2$

b $P \wedge Q$ = Mary is efficient and So is Ram
$T(P \wedge Q) = \text{Min}(T(P), T(Q))$
$= 0.65$

c $P \vee Q$ = Either Mary or Ram efficient
$T(P \vee Q) = \text{Max}(T(P), T(Q))$
$= 0.8$

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d) P -> q : If mary is efficient then so is Ram

I(P -> q) = Max (1 - T(P), T(q))
= Max (0.2, 0.65)
= 0.65

4) Write Predicate logic statements For given terms,

a) Ram likes all kinds of food.
b) Sita likes any thing which Ram likes
c) Raj likes those which sita and Ram both like
d) All likes some of which Ram likes

=> Let, Food(x) : x is a Food
likes(x, y) : x likes y

(a) $\forall_x$ Food(x) => likes(Ram, x)

(b) $\forall_x$ (likes(Ram, x) ^ likes(Ram, ^ likes(Sita, x)) => likes(Raj, x

(b) $\forall_x$ (likes(Ram, x)) => likes(Sita, x)

(d) $\exists_x$ (likes(Ram, x) ^ likes(All, x))